Phases of Stellar Evolution

Phases of Stellar Evolution

- Pre-Main Sequence
- Main Sequence
- Post-Main Sequence
- The Primary definition is thus what is the Main Sequence
  - Locus of “core” H burning
  - Burning Process can be either pp or CNO
  - ZAMS: Zero Age Main Sequence - locus of initiation of H burning

What is Happening?

- Pre-Main Sequence: Gravitational Collapse to ignition of H Burning
- Post-Main Sequence: Collapse of H exhausted core to final end

- THE DETERMINING AGENT IN A STAR'S LIFE IS GRAVITY

Virial Theorem

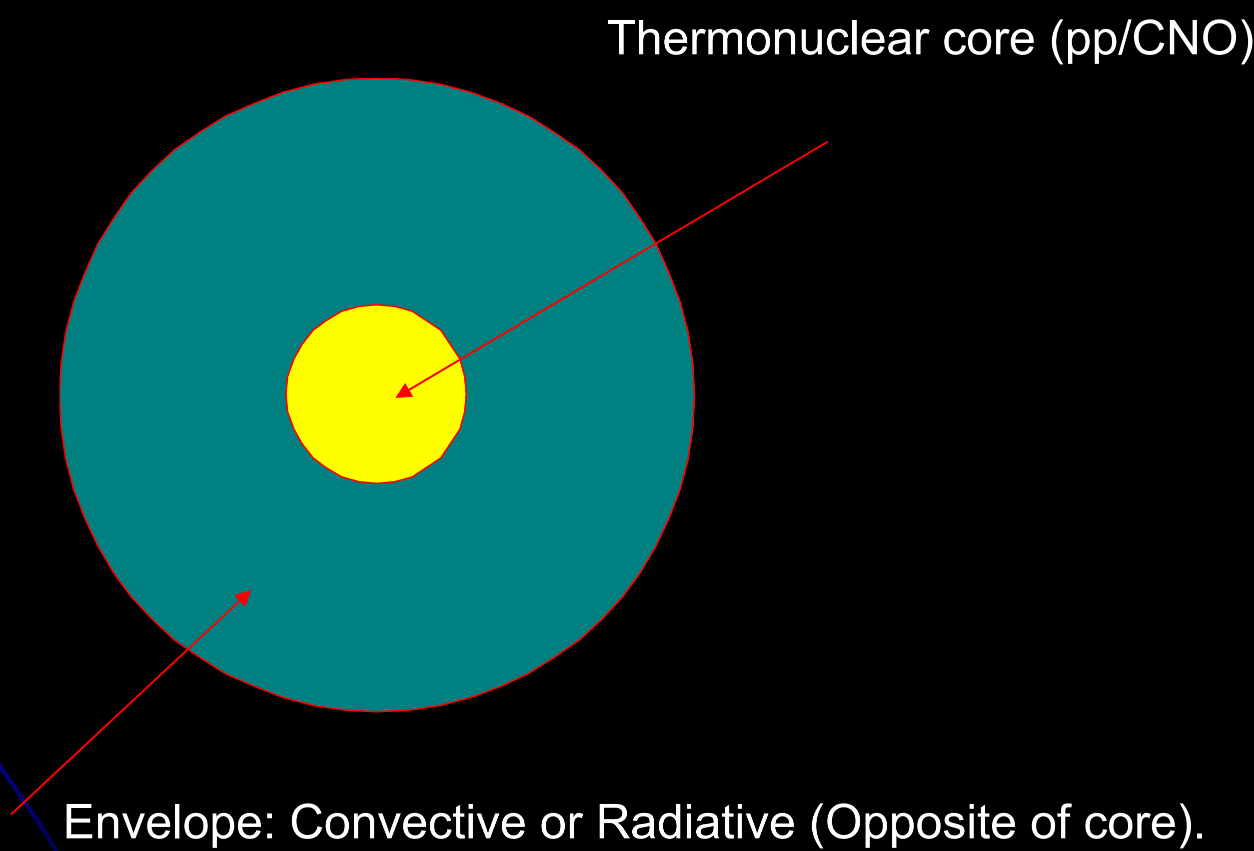
- $E = U + \Omega$ ---- Non-Relativistic Total Energy
- $2U + \Omega = 0$
  - U = Internal Energy
  - Ω = Gravitional Binding Energy
- Differential Form: $\Delta U = -1/2\ \Delta\Omega$
- ***Pre-Main Sequence*** is dominated by the Virial Theorem

General Principles of Stellar Evolution

- The initial effect of nuclear burning is to increase the mean molecular weight.
  - Hydrogen Burning: $4\,{}^{1}H \rightarrow {}^{4}He$ means $\mu \rightarrow 8/3\mu$
  - When μ increases the pressure is lowered and the core contracts, then T and P increase and thus hydrostatic equilibrium may be restored.
- When T increases, the temperature gradient increases. This causes an increase in luminosity (energy flow increases)
  - In order to balance energy generation and luminosity, the star must increase its luminosity (which does happen on the MS). As the temperature is rising the energy generation goes up and thus so does the luminosity.

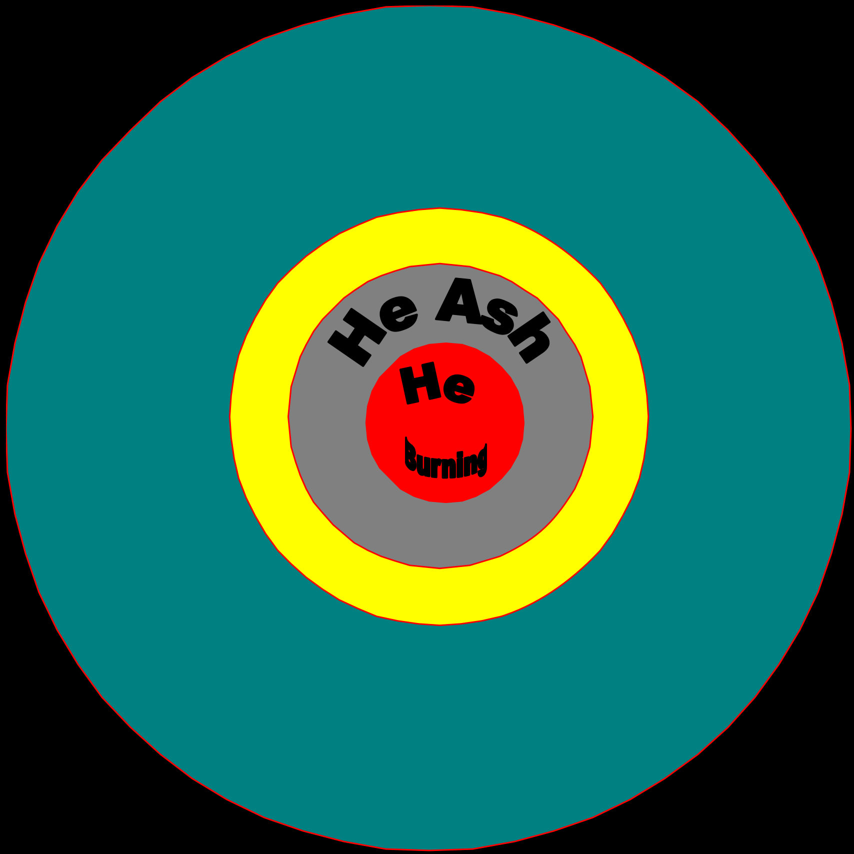
Main Sequence Configuration

The core is either convective (CNO burners) or radiative (pp burners)



Later Stage Configuration

- In a more massive star the rising core temperature will (might) cause ignition of higher mass nuclei. Outside is a region of processed material and outside that could be a region where the previous stage is still occurring. This is shell burning.



Evolution of A ProtoStar

We consider only the immediate Pre-Main Sequence

- Principle Constituents: H, H_2, He, (dust)
  - γ (c_p/c_v) is below 4/3: induced by the ionization of H, He, and the dissociation of H_2
  - When H and He are fully ionized $\gamma \rightarrow 5/3$ and the collapse becomes quasi-static
  - Virial Theorem says ½ of the energy of collapse goes into heating and ½ into radiation.
    - Bolometric Magnitude of a 1 pc cloud of radius 1 pc
      - $L = 4(206265*1.496(10^{11}))^2T^4 = 1.2(10^{34})\ 5.67(10^{-8})\ 100^4$
      - $L = 6.8(10^{34})$ J/s = 34000 $L_\odot$

Where Does the Energy Go?

- First Completely Ionize the He
  - E = (#He/gm)🕮(Mass Fraction)🕮(Ionization Energy)
  - E = $(N_0/4)$ 🕮 Y 🕮 E_{He}
  - $E_{He} = E_{He\ I} + E_{He\ II} = 78.98$ eV
- Similarly for H and H2
- $E_I = N_0 X E_H + ½\ N_0 X E_D + 1/4\ N_0 Y E_{He}$
- $= 1.9(10^{13})[1-0.2X]$
  - E_D = Energy of dissociation for H_2

Internal Energy

- From the Virial Theorem
- ME_I ☺ $\frac{1}{2}\,\alpha\, GM^2/R$ (M is the Mass collapsing)
  - α depends on the order of the polytrope
  - $n = 1.5$ ($\gamma = 5/3$) $\alpha = 6/7$
  - α is always of order unity
- Now solve for the radius:
  - $R/R_{\odot} = 43.2(M/M_{\odot}) / [1 - 0.2X]$
  - This is the maximum radius for a stable star at the beginning of its evolution
  - If larger then ionization and disassociation will not be complete.
  - Once $43.2(M/M_{\odot}) / [1 - 0.2X]$ is achieved quasistatic evolution is possible.

What is the Central Temperature?

- $T_c = 3(10^5)\mu(1\text{-}0.2X)$
- This is always less than ignition temperature (10^7K) so the energy source is gravitational collapse.
- E_T = Total Energy of the Star
- Luminosity = Energy Flow / Time = dE_T / dt

$$L = \frac{dE_T}{dt} = \frac{1}{2}\alpha\frac{d}{dt}\left(\frac{GM^2}{R}\right)$$

$$= -\frac{\alpha}{2}\frac{GM^2}{R}\frac{\dot{R}}{R}$$

Time Scale

$$L = -\frac{\alpha}{2}\frac{GM^2}{R}\frac{\dot{R}}{R}$$

- L is a positive number so R(dot) must be negative; ie, the star is contracting
- Time Scale $\propto \Delta E / L$
- $\Delta t = \Delta E / L \sim -1/2\ \Delta\Omega / L \sim GM^2/(2RL)$
- $\Delta t = 1.6(10^7)\ (M/M_\odot)^2(R_\odot/R)(L_\odot/L)$ years
- So for the Sun $\Delta t \sim 2(10^7)$ years
- At 10 $M_\odot$: $L \sim 10^5\ L_\odot$ and $R \sim 500\ R_\odot$ so $\Delta t \sim 32$ years!

Hayashi Track I

- Central Temperatures are low
- Opacities are large due to contributions from ionization processes (bf transitions!)
- Radiative processes cannot move the energy so convection dominates
- For an ideal gas T^3/ρ varies slowly (except in the photosphere) so the object is fully convective.
- The Hayashi Track is the path a fully convective contracting star takes in the HR diagram.

Hayashi Track II

- One can approximate the Hayashi Track as

  $$\text{Log } L = 10 \log(M) - 7.24 \log T_{eff} + \text{const}$$

- This is a very steeply descending function
- Why does L decrease?
  - Star is contracting and T_{eff} is increasing
  - $L \propto R^2T^4$ but R is decreasing very quickly and the radius decrease is dominating the luminosity
- For "low" mass stars a better approximation is $\log L = -38.7 \log T_{eff} - 6.74 \text{ Log } M + \text{const}$ which means the luminosity decreases very rapidly.

Later Phase Low Mass Protostars

- As the interior temperature increases the opacities change (bf gives way to electron scattering which means less opacity)
- A radiative core develops
  - Once the radiative core develops the core is not sensitive to the envelope. This yields the "constant" luminosity phase as the star moves to the left towards the MS.
  - For this phase $\log L = 0.8 \log T_{eff} + 4.4 \log M + C$

Pre-Main Sequence Tracks

Table 8.2
Elapsed time in years of selected points along evolutionary tracks leading to the main sequence, as shown in Figure 8.2.

	$M/M_\odot$								
Point	15.0	9.0	5.0	3.0	2.25	1.5	1.25	1.0	0.5
1	6.740×10^2	1.443×10^3	2.936×10^4	3.420×10^4	7.862×10^4	2.347×10^5	4.508×10^5	1.189×10^5	3.195×10^5
2	3.766×10^3	1.473×10^4	1.069×10^5	2.078×10^5	5.940×10^5	2.363×10^6	3.957×10^6	1.058×10^6	1.786×10^6
3	9.350×10^3	3.645×10^4	2.001×10^5	7.633×10^5	1.883×10^6	5.801×10^6	8.800×10^6	8.910×10^6	8.711×10^6
4	2.203×10^4	6.987×10^4	2.860×10^5	1.135×10^6	2.505×10^6	7.584×10^6	1.155×10^7	1.821×10^7	3.092×10^7
5	2.657×10^4	7.922×10^4	3.137×10^5	1.250×10^6	2.818×10^6	8.620×10^6	1.404×10^7	2.529×10^7	1.550×10^8
6	3.984×10^4	1.019×10^5	3.880×10^5	1.465×10^6	3.319×10^6	1.043×10^7	1.755×10^7	3.418×10^7	
7	4.585×10^4	1.195×10^5	4.559×10^5	1.741×10^6	3.993×10^6	1.339×10^7	2.796×10^7	5.016×10^7	
8	6.170×10^4	1.505×10^5	5.759×10^5	2.514×10^6	5.855×10^6	1.821×10^7	2.954×10^7		

SOURCE: I. Iben, Jr., *Astrophys. J.*, **141**:993 (1965).

Tracks by Icko Iben

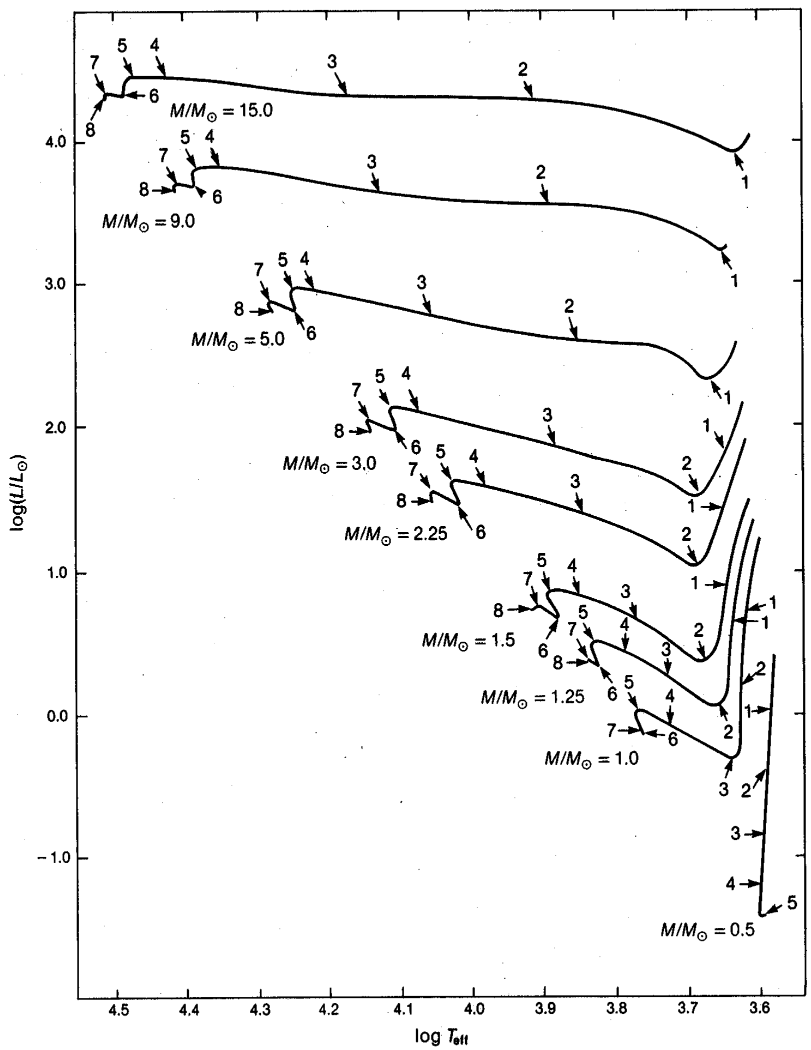


Figure 8.2. Evolutionary tracks for stars before the main sequence. The time intervals required for stars to reach the numbered points are given in Table 8.2. The locus of the terminal points of the paths defines the zero age main sequence.

A 1 Solar Mass Time History

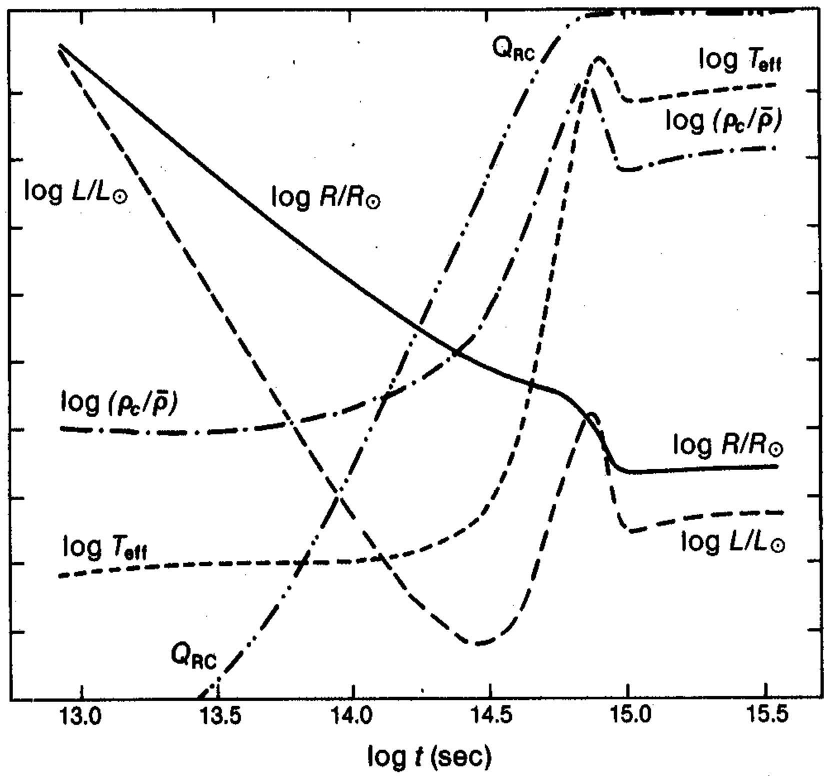


Figure 8.3. Time-variation of stellar parameters for a 1 $M_\odot$ star as it evolves down the Hayashi track; the time is in seconds. The curve Q_{RC} gives the mass fraction in the radiative core, with the ordinate scaled such that $0 \leq Q_{RC} \leq 1.0$. For other curves, $3.58 \leq \log T_{eff} \leq 3.78$; $0 \leq \log(\rho_c/\bar{\rho}) \leq 2.0$; $-0.4 \leq \log(L/L_\odot) \leq +0.6$; and $-0.4 \leq \log(R/R_\odot) \leq +0.6$.

NGC 2264

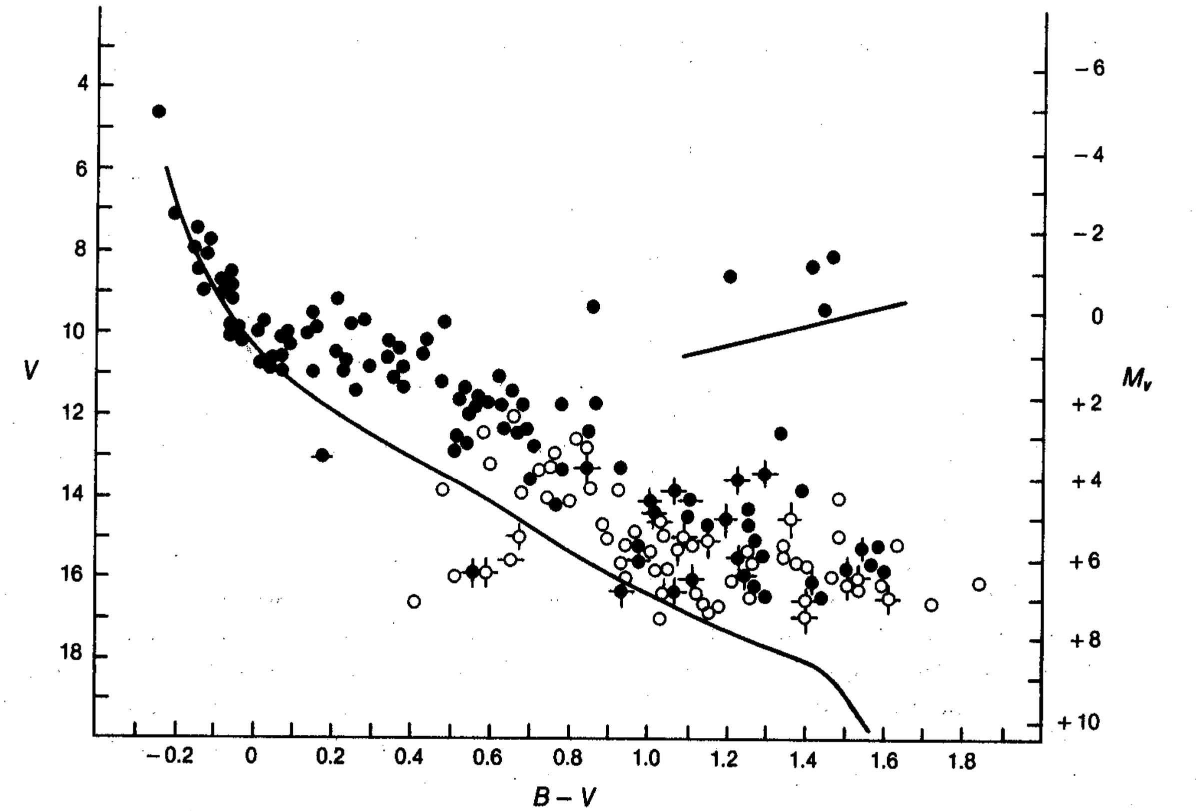


Figure 8.4. Color-magnitude diagram of the young galactic cluster NGC 2264. The lines represent the main sequence and the giant branch, corrected for uniform reddening. Points with horizontal bars denote stars with H_α line emission; those with vertical bars denote variable stars.

Evolutionary Tracks

- The evolution of a star is given by evolutionary tracks.
  - They give the position of the star in HR diagram as it moves in temperature and luminosity.
  - The amount of time between successive (T,L) points is variable and depends on the mass.
- An isochrone is (T,L) for a sequence of masses at a fixed time plotted in the HR diagram.

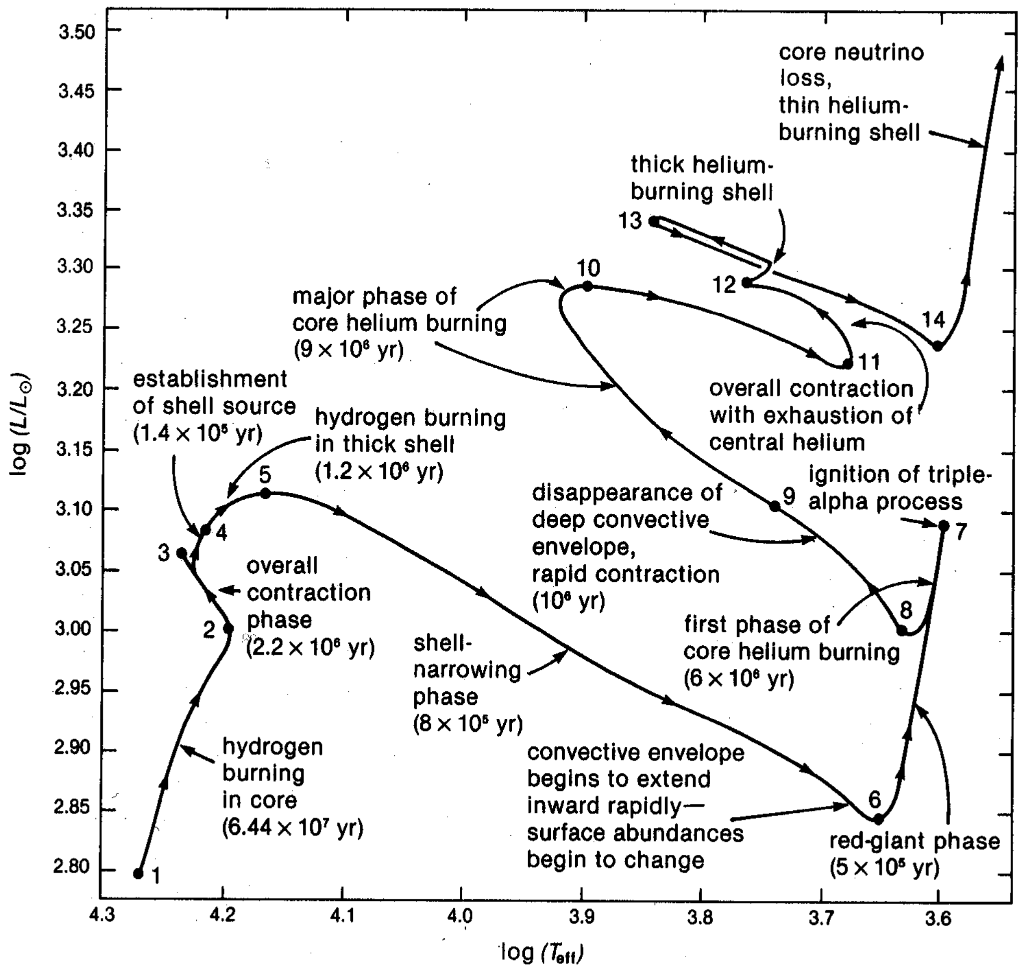


Figure 8.1. Evolutionary path of a Population I star of 5 $M_\odot$ in the Hertzsprung-Russell diagram, showing processes characterizing each stage. Time in parentheses is the duration of the stage between the numbered points. Luminosity is in $L_\odot$ and temperature in K. The elapsed time between points 10 and 11 is 10^6 years.

A 5 M Track by Icko Iben, Jr.

A Movie

- Rant about QuickTime Here.
- Then Show the Movie.

Evolutionary Sequences

- Stellar evolution is more difficult than stellar structure.
  - Structure is static but by its nature evolution is dynamic.
  - Some of the changes take place on free-fall timescales.
  - The structure and its rate of change depend on the previous structure.
  - The problem becomes one of choosing time steps that are sufficiently small with respect to the rate of change, yet practical from the point of view of computer time.

Stellar Structure Equations

Time Dependent Versions

- Hydrodynamic Equilibrium

$$\frac{\partial P}{\partial r} = -\frac{Gm\rho}{r^2} - \rho \frac{d^2 r}{dt^2}$$

- Mass Continuity

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho$$

- Energy Flow

$$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi ac}\frac{L(r)}{r^2 T^2}$$

- Energy Generation

$$\frac{\partial L}{\partial r} = 4\pi r^2 \rho(r)\left[\varepsilon(r) - T\frac{dS}{dt}\right]$$

Remember

- The following are functions of r
  - P, T, ρ, m, κ, L, ε
- Note that equation 3 (energy flow) contains all of atomic physics in κ!
- Equation 4 has all of nuclear physics in ε!
- Thermodynamics is in 1 & 4: T (dS/dt) is the energy of collapse expressed in terms of the entropy change.

The Main Sequence

ZAMS: Zero Age Main Sequence

- Note that the observational MS has a finite width due to the admixture of ages.
- As a star evolves on the MS it evolves up in luminosity and down in T
- The dividing point for the energy generation cycles occurs at about $2(10^7)$ K
  - $< 2(10^7)$ K uses pp with radiative core and convective envelope
  - $> 2(10^7)$ K uses CNO with convective core and radiative envelope.

Consider the CNO Cycle

- For $1.2(10^7)$ K < T < $5(10^7)$ K the energy generation goes as $20 \gtrsim n \gtrsim 13$ in $\varepsilon = \varepsilon_0 \rho T^n$
- This means that the star will develop a very large temperature gradient due to the sensitivity of the energy generation to T
  - To see this: $\partial L/\partial r \propto \varepsilon$ and $\partial T/\partial r \propto L$
- This also means these stars have a highly centralized core in terms of energy generation: a 2% decrease in T yields a 33% decrease in energy generation (n=20)
- Large temperature gradients means convection which dominates CNO cores

For the pp Cycle

- For the pp cycle at $4(10^6) < T < 2.4(10^7)$ K
  - $6 \gtrsim n \gtrsim 3.5$ in $\varepsilon = \varepsilon_0 \rho T^n$
  - This means that the temperature gradient is much smaller. There is less centralization in the energy generation and little tendency for convection in the core.
  - pp cores are radiative

Envelope Structures

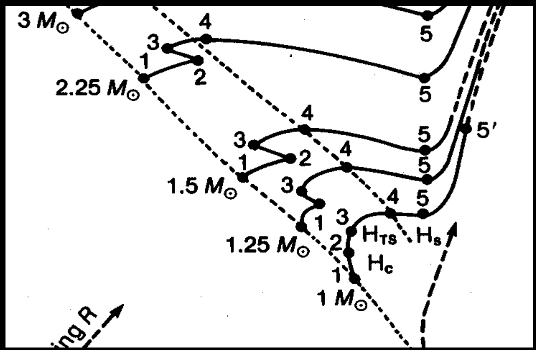
- Reverse of the core structure
- "Low" mass stars have convective envelopes instigated by "large" H and He ionization zones. Note that μ changes dramatically in an ionization zone and they are intrinsically unstable.
- "High" mass stars have "shallow" ionization zones which do not perturb the structure as much.

The ZAMS

- At the time of entry onto the MS the core temperature is sufficient to initiate burning:
  - pp starts
  - $^{12}C \rightarrow {}^{14}N$ by 2 proton captures
  - This happens competitively with pp at the initial core temperature
  - Leads to an equilibrium configuration of CN cycle ^{14}N
  - We at once convert all ^{12}C to ^{14}N but to continue the CNO process we must do $^{14}N(p,\gamma)\,{}^{15}O$. This is very slow and stops the CN processing if $T < T_{crit}$ for the CNO cycle.
- Why is this important?
  - Because of the T sensitivity. For $^{12}C \rightarrow {}^{14}N$ $\varepsilon = \varepsilon_0 \rho T^{19}$
  - The core becomes convective!

Why does L Drop?

- The core previous to $^{12}C \rightarrow {}^{14}N$ was radiative.
- It becomes convective which is centrally condensed, halts contraction, and <u>does work</u> against gravity.
  - Energy goes into work not luminosity - 80% in fact.
- After ^{12}C burns there are two possibilities:
  - In a low mass star a radiative core is reestablished due to pp domination
  - In a high mass star CNO dominates and a convective core remains.
    - Fresh ^{12}C from convection or 3α



Evolution on the Main Sequence

Really Slow!

- Timescale: $t_n \sim mc^2/L$
  - For the Sun $t_n \sim mc^2/L \sim 2(10^{33})\ 9(10^{20})\ /\ 4(10^{33})$ s
  - This is about $1.4(10^{13})$ years for complete conversion so the process does not need to be efficient!
- One can assume that static structure equations will hold.
- As time passes there will be chemical evolution in the core through nuclear burning.
- Augment the static structure by time dependent burning.

Burning Hydrogen

- Assume X (protons) and Y (alpha particles) are the only species.
  - X,Y specify a static model at any time
  - The time rate of change of (X,Y) are then related to energy generation rates and the energy release per gram of matter.
- For X: Reduction is by pp, CN, or CNO
  - Let us find dX/dt
  - For the pp chain Q = 26.73 MeV for each 4 H converted.
  - $E_{pp} = Q_{pp} / 4m_H$ = energy / gram

What is dX/dt?

$E_{pp} = Q_{pp} / 4m_H$ = energy / gram

- But what what we want is dX/dt which has units of gram / s = (energy / s) 🕮(gram / energy)
  - ε_{pp} has units of energy / s
  - E_{pp} has units of energy / gram
- For pp only $dX/dt = -\varepsilon_{pp}/E_{pp}$
- pp + CN: $dX/dt = -\varepsilon_{pp}/E_{pp} - \varepsilon_{CN}/E_{CN}$
  - Low Mass: $T < 2(10^7)$K pp dominates
  - High Mass: $T > 2(10^7)$K CN dominates
    - E_{pp} and E_{CN} are constants
    - ε_{pp} and ε_{CN} depend on the structure (T and ρ)

dY/dt

- The sink is 3α, the source is dX/dt
  - Note that dX/dt is intrinsically negative
- $dY/dt = -3\varepsilon_\alpha/E3_\alpha - (1/4)dX/dt$
  - $4H \rightarrow 1\ He$
- This is for a static zone; that is, a radiative low mass core.

Core Convection

- Time Scale for convection is of order months
  - Instantaneous with respect to the time scale of the reactions
- This means that for a convective zone there is an "average" composition: X_c, Y_c, (Z_c)
  - For a correct treatment we must consider the intrusion of the convection into the radiative zone but neglect this for now.
- The rate of change of X_c is ε/E (per process) averaged through the zone.
  - For pp: dX_c/dt = -ꝏε_{pp} /E_{pp} $dM/\Delta m$ integrate between m_1 and m_2 and $\Delta m = m_2 - m_1$. Note that the mass of the convective zone $\neq$ mass of the core (necessarily)

Process to Calculate a Sequence

- Assume X,Y: calculate structure
- Estimate ΔX, ΔY: dX/dt 📖 Δt where dX/dt is the instantaneous rate and Δt is the time step.
- The composition at $t_0 + \Delta t$ is then $X = X + \Delta X$ and $Y = Y + \Delta Y$.

The Lower Main Sequence

To Reiterate

- Energy generation by pp chain
- $T_c < 2(10^7)$ K
- M $\lesssim$ 2 Solar Masses
- Radiative cores and convective envelopes
- Core size decreases with total mass
- Core is initially homogeneous
- Energy generation rate: $\varepsilon = \varepsilon_0 \rho X^2 T^{3.5 \text{ to } 6}$

Evolution on the MS

$\varepsilon = \varepsilon_0 \rho X^2 T^{3.5 \text{ to } 6}$

- Note the X^2 dependence in ε. As X decreases so does ε unless T or ρ increase.
- If ε decreases then so does P
  - Contraction follows: Virial Theorem allocates ½ the resulting energy to radiation and ½ to heating.
  - This means ρ increases (contraction) and T increases (Virial Theorem)
    - ε increases
    - Slight increase in core radius and envelope
    - L will also increase
      - T_{eff} will not increase much due to increase in R

At the Solar Age

T = 4.3 Gigayears

- Note that 90% of the mass r/R = 0.5.
- X normalized to 1: depletion limited to r/R < 0.3 (0.5 in m/M)
- $L = L_{\odot}$ at r = 0.3R.

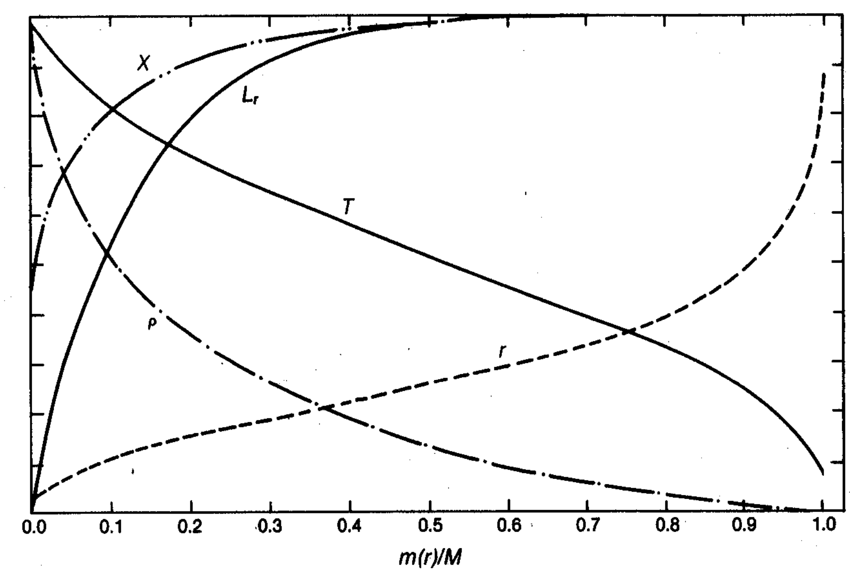


Figure 9.2. A 1 $M_{\odot}$ model during main-sequence hydrogen burning at time 4.2699×10^9 years (between points 1 and 2 in Figure 9.1), showing radius, density, temperature, total luminosity, and hydrogen abundance versus mass fraction. The lower limits of the ordinate are zero. The upper limit for each curve is: $r = 0.9683\ R_{\odot}$, $\rho_c = 159.93$ g cm^{-3}, $T_c = 1.591 \times 10^7$ K, $L = 1.0575\ L_{\odot}$, and $X_c = 0.708$; $P_c = 2.5186 \times 10^{17}$ dynes cm^{-2}. The elapsed time is measured from the initial model for the phase before the main sequence.

Just Before Leaving

- r < 0.03R is isothermal He.
- H burning: 0.03 < r < 0.3 R
- ε = dL/dm is just the slope of L. ε is now large with respect to previous values.
- The development of an inhomogeneous structure marks the end of the MS in this mass range.

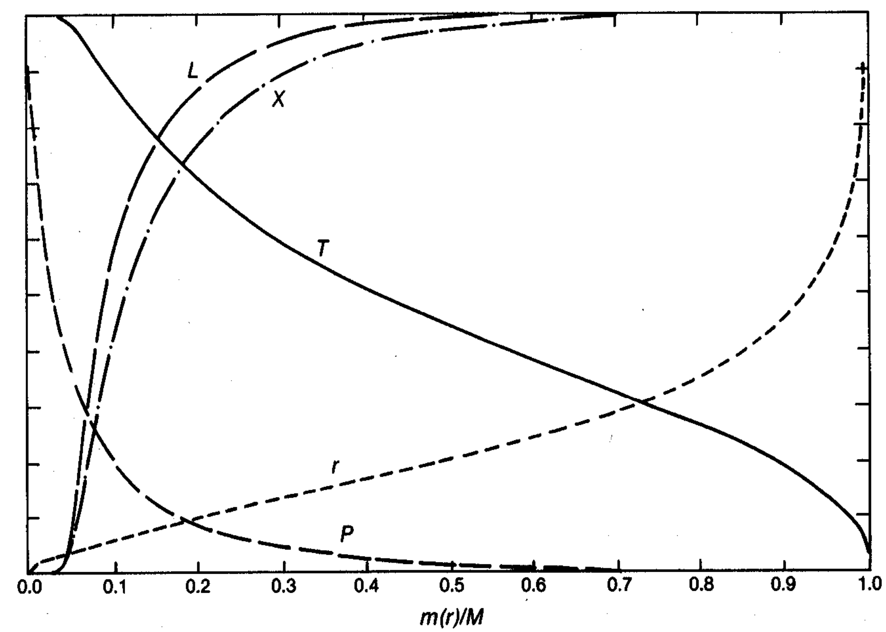


Figure 9.3. Same as Figure 9.2 for 1 $M_\odot$ model between points 2 and 3 of Figure 9.1. The elapsed time since the initial model for the phase before the main sequence is 9.2015×10^9 years. The lower limits of the ordinate are zero. The upper limit of the ordinate for each curve is $r = 1.268\ R_\odot$; $P_c = 1.315 \times 10^{18}$ dynes cm^{-2}; $T_c = 1.91 \times 10^7$ K; $L = 2.13\ L_\odot$; $X_c = 0.708$. The actual stellar radius is $R + 1.353\ R_\odot$, and the central density is 1026.0 g cm^{-3}.

Upper Main Sequence

$T_c > 2(10^7)$ K

- H burning by the CNO cycles
- Convective cores
  - Homogeneous evolution in the core even though H burning is more rapid in the center than outer edges.
  - Opacity: Kramer's 2-3 Solar Masses
  - Electron Scattering > 3 solar masses
- $\varepsilon = \varepsilon_0 X Z_{CNO} T^n$
  - Since ε goes as X the generation rate is not so sensitive to composition changes
  - This means the core contraction brought on by H depletion is not as severe as on the lower main sequence

High Mass Evolution

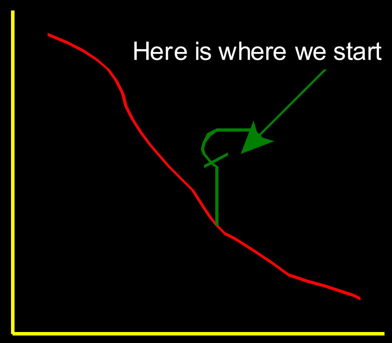
- As the mass increases
  - R, L, T_{eff}, and T_c all increase but ρ_c does not.
  - If the opacity is electron scattering then there is a smaller dependence of luminosity and T_{eff} on mass.
- Note that the main difference between high mass and low mass evolution is that high mass stars do not form thick burning shells about the He core as the star ages on the MS. In fact, He cores do not form until "all" H burning ceases.
  - This is due to convection homogenizing the core.
  - So burning merely continues until X reaches about 0.05 when ε falls below the amount needed to support the core.

The Isothermal Core

Low Mass Stars Only

- $L(0) = 0$ and $dL/dr \approx 4\pi r^2 \rho \varepsilon(r)$
- If $\varepsilon = 0$ throughout a region then $L = 0$ as well.
- But $dT/dr \sim L(r)$ so if $L = 0$ then $T = \text{Constant}$
- So what supports the core (it is NOT degenerate)? A steep density gradient.
- There is a limiting mass for this case - it is called the Schönberg-Chandrasekhar limit and is approximately 0.12 solar masses.
  - A core with $M_c < M_{SC}$ is stable but a burning shell above it will continually add mass.
  - The result is that the limit will be exceeded.
  - The core will start to collapse

Termination of the MS



- Shell source: $0.1 < m(r)/M < 0.3$
- L increases
- Star expands.
- T should go up but R increases to such an extent that T actually falls.
- This takes about 12% of the MS lifetime.
- Eventually the core mass exceeds the Schönberg-Chandrasekhar limit (in all but the lowest masses) and the core is forced to contract. The MS is over.

High Mass: M > 1.25 Solar Masses

- No isothermal cores are formed
- X decreases to about 5% of its original value and core shuts down.
- Contraction starts
- L increases
- T_{eff} increases initially
- The contraction induces a shell to start. The MS is over.
- The shell provides L
- The core contracts
- The envelope expands: T must decrease.

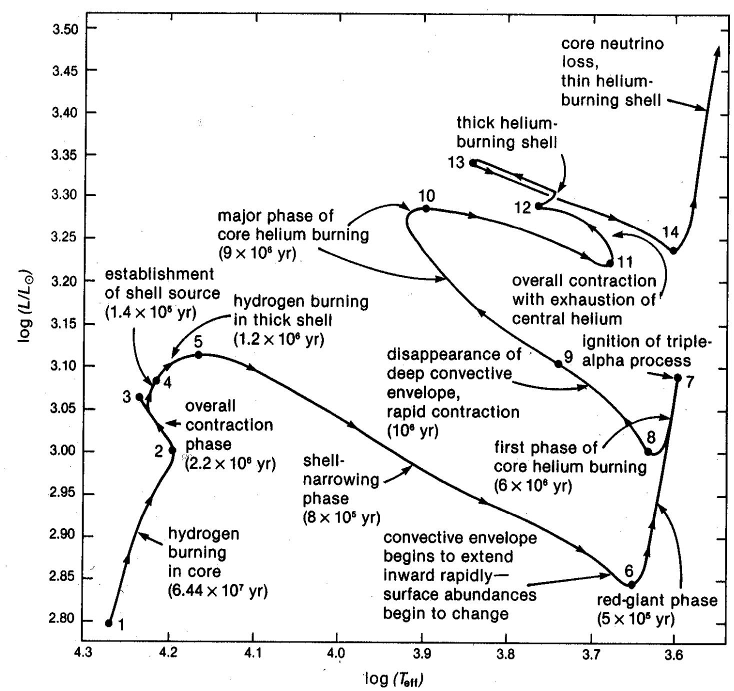


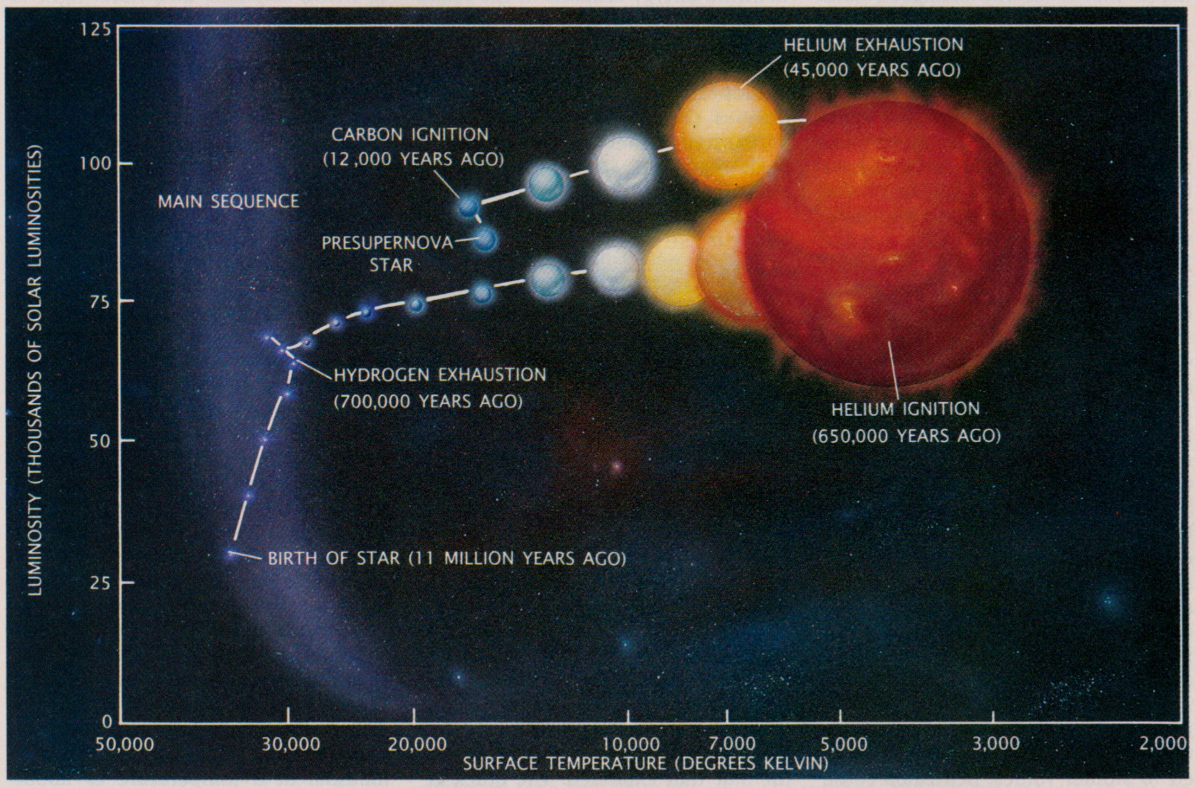
Figure 8.1. Evolutionary path of a Population I star of 5 $M_\odot$ in the Hertzsprung-Russell diagram, showing processes characterizing each stage. Time in parentheses is the duration of the stage between the numbered points. Luminosity is in $L_\odot$ and temperature in K. The elapsed time between points 10 and 11 is 10^6 years.

Post-Main Sequence Evolution

A Dramatic Series of Events

- Events are less certain as the possibilities become wider in scope
  - Dynamical Effects.
  - Observational data more limited but what is known agrees rather well with the theory especially in single stars.
- One cannot make certain events happen numerically ab initio
  - Pulsation in Variables
  - Ignition of 3α in a “controlled” fashion , esp low mass
  - SN dynamics
  - Deflagrations are a problem
  - Binary evolution

Tracks: Single Stars with No Mass Loss



HISTORY of the progenitor star began some 11 million years ago on the "main sequence"—the region hydrogen-burning stars (most stars in the sky) occupy on graphs of luminosity versus surface temperature. After about 10 million years the hydrogen in the star's core was burned to helium, and the core contracted and became hotter. In response the star's envelope expanded and cooled, and the star moved to the right, off the main sequence. As the core got hot and dense enough to burn helium, the star bloated into a red supergiant, with a cool envelope several times the size of the earth's orbit. After helium was exhausted, the envelope contracted and heated up again, and the star became a blue supergiant. In that form it burned successively heavier elements, ultimately making the iron core whose collapse triggered the supernova. The scenario is based on the authors' calculations for an 18-solar-mass star with a starting composition typical of the Large Magellanic Cloud.

What Do We have to Add?

- The most important new feature to allow for is chemical inhomogeneities and associated shell sources.
- Active Shells: Currently burning
- Inactive Shell: Chemical inhomogeneity
- The behavior of expansions and contractions change over active shells. Volume changes reverse over active shells.

What Do Inhomogeneities Do?

- Inhomogeneous composition leads to greater central condensation
- The position of a burning shell remains fairly constant in radius.
- Volume changes (contraction, expansion) reverse at each nuclear burning shell, but remain unaffected by the presence of inactive shells.

Desidera

- Nuclear ash has (usually) $\mu > \mu_{fuel}$
- Larger μ ==> larger central core values for ρ
- Note that the core $\gamma = 4/3$ $(n = 3)$.
- Stationary shells:
  - If it tries to burn inwards then T increases and so does ε which means that P will increase forcing the shell back out. Also trying to burn inwards means that one is moving to zones depleted in the current fuel.
  - Cannot go out (in radius) because T will be too low to support the burning.

Volume Reversals

VIP

- If one has a contracting core with an active shell ==> envelope expansion.
- If one has a contracting core with two active shells ==> envelope contraction.
- Why? Consider the following (long) argument.

Central Condensation

- A measure of the central condensation for a particular volume is the ratio $\rho(r)$ (the local value of the density) to $\langle\rho(r)\rangle$ the mean density of the material interior to r.
- We define $U \equiv 3\rho(r)/\langle\rho(r)\rangle$ where $\langle\rho(r)\rangle = 3m(r) / 4\pi r^3$
- One can show: $U = d(\ln(m(r))/d(\ln(r)) = d \ln(q) / d \ln(r)$ where $q \equiv m(r)/M$
- What are the limits on U?
  - At $r = R$ $\rho(r) = 0$ $U = 0$ at the upper boundary
  - At $r = 0$ $\rho(r) = \rho_c$ and $\langle\rho(r)\rangle = \rho_c$ $U = 3$

Consider the Interface

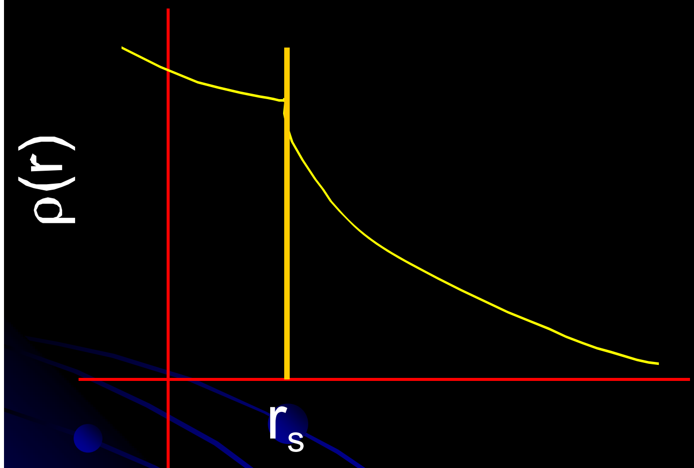
μ_c, ρ_c μ_e, ρ_e

- Core Envelope interface with no burning
- If the star is to be stable T and P must be continuous across the interface.
- Assume the ideal gal law: $P = nkT = \rho kT/m_H$ which means $PV / T = P'V' / T'$
- Now rearrange using equal volumes or $V = V'$ so $P/T = P' /T'$ or $\rho/\mu = \rho' /\mu'$ or in terms of our interface $\rho_c/\mu_c = \rho_e/\mu_e$.
- Now $\mu_c > \mu_e$ so $\rho_c > \rho_e$
- If the boundary is sharp: $<\rho(r)> =$ constant

Interfaces

- Note that if there is a $\rho(r)$ discontinuity then there is a $U(r)$ discontinuity
- Since $U \equiv 3\rho(r)/\langle\rho(r)\rangle$ and $\langle\rho(r)\rangle$ = constant for a sharp interface U goes as ρ. This means since $\rho_c/\mu_c = \rho_e/\mu_e$ that $U_c/\mu_c = U_e/\mu_e$.
- There are two additional characterizations:
  - $V \equiv -d \ln P / d \ln r$ (Pressure scale height)
  - $N+1 \equiv d \ln P / d \ln r = -V$
  - These can be evaluated adiabatically and if done that way N is related to Γ_2
  - An adiabatic process is one in there is no heat flow
    - A free expansion is an adiabatic process
      - No Heat Flow
      - No Work

Characterisitics of Shell Sources



- Gas is ideal: $\rho = KT^n$
- $dP/dr = dP/dT\ dT/dr$

  $= dT/dr\ (d/dT\ (\rho kT/\mu m_H))$
- $= k\rho(n+1)/\mu m_H\ dT/dr$
- dP/dr can be specified by the hydrostatic equation
- m_c is concentrated near r_s

Shell Sources II

- m(r) ☺ m_c for r >> r_s
- $T_s = (\mu m_H G m_c/(k(n+1)))\ (1/r_s) + \text{const}$
- Note that Ts ~ $1/r_s$
  - No Motion of the shell
  - For Example, in a 1 M📪 star the shell location is at R ~ $0.03R_\star$

Volume Changes and Shell Structure

$$\ln(R) = \ln(r_s) + \int_{q_s}^{1} \frac{d \ln q}{U}$$

- R = Stellar Radius
- $q_s = m_c/m$ (mass fraction in core)
- r_s = constant (so the stellar radius depends on the integral)
- $U = 3\rho(r)/\langle\rho(r)\rangle$

What Happens?

- Consider ρ_c increasing due to core condensation
- r_s remains fixed.
- ρ at the edge of the shell (inside) will decrease
- $<\rho(rs)>$ is constant (to first order : $\Delta M = 0$)
- Therefore we have a decrease in U_c but if U_c decreases so must U_e
- Therefore: reduce U(r) in the lowest levels of the envelope where 1/q is largest ==> decrease the density.

A Practical Statement

- As the central condensation grows the density near the bottom of the shell decreases and to maintain continuity the envelope responds by expanding (above the burning shell).
- Note that in the case of two burning shells one gets envelope expansion!