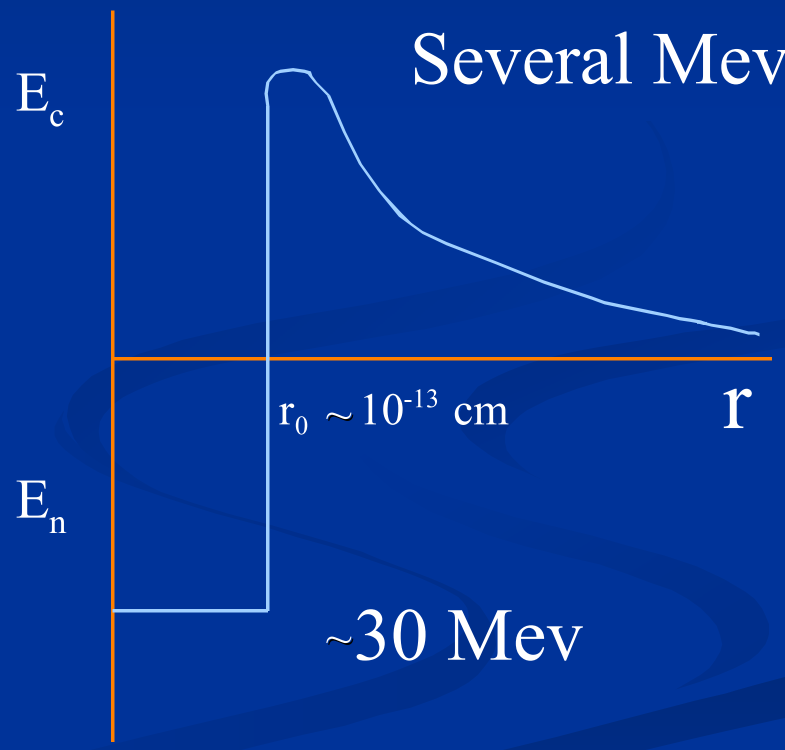
Shall We Play a Game of Thermonuclear War?

Thermonuclear Energy Sources

- Electrostatic Potential
  - $E_c \sim e^2/r_0 \simeq 1.7(10^{-6})$ ergs $\simeq$ 1 Mev
- In a star pressure and high temperature cause penetration of the Coulomb barrier. Or do they?



The Solar Core

- The velocities are Maxwellian:
- $f(E) = (2/\pi^{1/2})\ (1/kT)^{3/2}\ e^{-E/kT}\ E^{1/2}$
  - E is the average thermal energy of a proton
  - $\langle E \rangle = 3/2\ kT$ which is several keV at $2(10^7)$ K or $10^{-3}\ E_c$!
- The Sun does not have sufficient numbers of protons at high temperatures to fuel its energy output! Or does it?

Barrier Penetration

- Consider $^{84}Po_{212}$: it is an α emitter with a laboratory half-life of $3(10^{-7})$ sec.
  - The particle has an energy of ~ 9 MeV. This particle should not be able to escape from a classical potential!
- But atoms are NOT classical systems.
  - They are quantum mechanical entities
  - The particles in a nucleus are described by a spatial probability distribution.
  - The probability density is highest where the classical theory says the particle should be but the probability is non-zero at *every* other point.
  - Sometimes the α particle will find itself at $r > r_0$!

The Inverse Problem

- We are interested in the inverse of the Po case.
  - We want to break the barrier coming in.
- Classically there is no opportunity for the penetration but in QM there is a finite probability of the event; that is, the incoming particle will sometimes find itself at $r < r_0$.
  - The closer you can get the incoming particle to the nuclei to be penetrated the better the probability.
  - Thus penetration is favored in a fast moving particle.

Miniumum Mass for Stars

Barrier Potential $Z_1Z_2e^2$

- "Low Temperatures" favor penetration of low Z species
  - Low mass objects have lower values of T
    - Only H can be burnt
- Core Temperature $T_c > T_I$ (ignition) for a particular fuel
- If we have a uniform density: $\rho = M/4\pi R^3$
- $T_c \simeq (Gm\mu m_H) / (5kR)$ (Hydrostatic Ideal Gas)

The Condition for Burning

- At high density matter becomes degenerate and the thermal energy of a photon must be less than the Fermi energy:

$$kT < \varepsilon_{F,e} = \frac{p_{F,e}^2}{2m_e} = \frac{\hbar^2 (3\pi n_e)^{3/2}}{2m_e}$$

- Charge neutrality must be maintained: $n_e = n_p = \rho / m_H$ from which one may obtain:

$$\frac{m_H}{\rho} > \frac{\hbar^3}{(2m_e kT)^{3/2}}$$

The Minimum Mass

- The Inter-nuclear Separation is:

$$\left(\frac{m_H}{\rho}\right)^{\frac{1}{3}} > \frac{\hbar}{(2m_e kT)^{1/2}}$$

  - Note that $2m_e kT$ is the thermal wavelength of an electron
- One can then obtain:

$$\mu^{\frac{3}{2}}\left(\frac{M}{M_{Sun}}\right) > 2.9(10^{-7})T^{\frac{3}{4}}$$

- For H ($\mu = ½$) at $T = 10^7$: $M/M_{\odot} > 0.14$
- A more detailed argument says: $M/M_{\odot} > 0.05$ to 0.08

Energy Release

The Process is characterized by $\varepsilon = \varepsilon_0 \rho^a T^n$

- Units: ergs gm^{-1} sec^{-1}
- ε_0: Constant depending on the reaction
- a: A constant which is ~ 1
- n: 4 for H burning (pp cycle) and 30 for C burning

Reaction Networks

- Energy release: photons ($h\nu$) + neutrinos
  - Only the photons contribute to the energy generation
  - Neutrinos are lost to the system
  - Note that the photon generation can be after the reaction: specifically in ee^+ annihilation
- A reaction network is a specific set of nuclear reactions leading to either energy production and/or nucleosynthesis.
  - pp hydrogen burning is one such network
  - CNO hydrogen burning is another
  - 3α (He burning) is another

The pp Chain

Process: PPI

- Initial Reaction: $p + P \rightarrow D + e^{+} + \nu_e$
  - Energy = 1.44207 MeV
  - Only D remains as the position undergoes immediate pair annihilation producing 511 KeV. The e is lost energy to the system.
  - This can be written: $H^1\ (p,\beta^{+}\nu_e)\ D$
- Next: $p + D \rightarrow He^3 + \gamma$
  - Produces 5.49 MeV
  - At $T < 10^7$ K this terminates the chain

PPI continued

- Lastly (at $T > 10^7$ K): $He^3 + He^3 \rightarrow He^4 + 2p$
  - This produces 12.86 MeV
- Total Energy for PPI: 2(1.18 + 5.49) + 12.86 = 26.2 MeV
  - Reaction 1 & 2 must occur twice
  - The neutrino in reaction 1 represents 0.26 MeV

PPII

He^4 must be present and $T > 2(10^7)$ K

- $He^3 + He^4 \rightarrow Be^7 + \gamma$ 1.59 MeV
- $Be^7 + p \rightarrow B^8 + \gamma$ 0.13 MeV
- $B^8 \rightarrow Be^8 + e^+ + \nu_e$ 10.78 MeV (7.2 in ν_e)
- $Be^8 \rightarrow 2He^4$ 0.095 MeV
- The total energy yield is 19.27 MeV (including the first two reactions of PPI.
- Note that when Be or B are made they end up as He.

PPIII

A Minor Chain as far as probability is concerned.

- Start with the Be^7 from PPII
- First reaction is $Be^7 + e^- \rightarrow Li^7 + \nu$
- Next $Li^7 + p \rightarrow 2He^4$
- This is the infamous neutrino that the Davis experiment tries to find.
- Note that we cannot make Li either!

Timescales for the Reactions

Reactants	Halflife (Years)
p + p	$7.9(10^{9})$
p + D	$4.4(10^{-8})$
He^{3} + He^{3}	$2.4(10^{5})$
He^{3} + He^{4}	$9.7(10^{5})$
Be^{7} + p	$6.6(10^{1})$
B^{8} and Be^{9} decay	$3(10^{-8})$

Nucleur Reaction Rates

- Energy Generation Depends on the Atomic Physics only (On A Per Reaction Basis)
  - Basics: $4H \rightarrow He^4$
    - $4H = 4(1.007825\ \text{AMU}) = 6.690594(10^{-24})$ gm
    - $He^4 = 4.00260\ \text{AMU} = 6.645917(10^{-24})$ gm
- ❍Mass = $4.4677(10^{-26})$ gm
  - $E = mc^2 = 4.01537(10^{-5})$ ergs = 25.1 MeV (vs 26.2 MeV ?!)
- Mass Excess ≡ ❍M = $M - Am_H$
- Reactions can be either exothermic or endothermic
  - Burning Reactions that produce more tightly bound nuclei are exothermic; ie, the reactions producing up to the iron peak.

Conservation Laws

- Electric Charge
- Baryon Number
  - $p = n = 1$
  - $e = \nu = 0$
- Spin

The Units in $\varepsilon = \varepsilon_0 \rho^a T^n$

- ε = ergs g^{-1} s^{-1} = g cm^2 s^{-2} g^{-1} s^{-1} = cm^2 s^{-3}
- Assume a = 1 $\therefore$ $\rho^a = \rho$: g cm^{-3}
- So cm^2 s^{-3} = ε_0 g cm^{-3} K^n
- ε_0 = (cm^2 s^{-3})/(g cm^{-3} K^n) = (cm^2/g) cm^3 s^{-3} K^{-n}
- So how do we get ε_0?
  - Bowers & Deming Section 7.3
  - Clayton

Reaction Cross Section

- Cross Section σ
  - Let X = Stationary nucleus
  - Let a = Moving nucleus
- Probability of a reaction per unit path length is $n_X\sigma$ where n_X is the density of particle X
  - $n_X\sigma$ = # cm^{-3} cm^2 = # cm^{-1} (This is 1 / MFP)
    - MFP = $(1 / n_X\sigma) = vt$ where t = time between collisions and v = speed of particles (the one(s) that are moving).
    - $t = 1 / n_X\sigma v$ or Number of Reactions (per Stationary particle/s) = $n_X\sigma v$
- Total number of reactions: $r = n_a n_X v\sigma(v)$
  - $r = cm^{-3}\ cm^{-1}\ cm\ s^{-1}$ = # / (cm^3 s)
  - The cross section must be a function of v

Integrate Over Velocity

- There actually exist a range of velocities f(v) so we must integrate over all velocities:

$$r = n_a n_X \int v\sigma(v) f(v) dv \equiv n_a n_X < \sigma v >$$

- The total energy is then rQ/ρ.
  - r = reaction rate in # / (cm^3 s)
  - Q = energy produced per reaction
  - ρ = density g cm^{-3}
  - rQ/ρ = cm^{-3} s^{-1} g^{-1} cm^3 ergs = ergs s^{-1} g^{-1} = $\varepsilon = \varepsilon_0 \rho^a T^n$

Penetration Factor

- The penetration factor is defined as:

$$e^{-4\pi^2 a Z_1 Z_2 \left(\frac{mc^2}{2E}\right)^{\frac{1}{2}}} \equiv e^{\frac{-b}{E^{1/2}}}$$

$$a = \frac{e^2}{hc}$$

$$b = \left[-4\pi^2 a Z_1 Z_2 \left(\frac{mc^2}{2}\right)^{\frac{1}{2}}\right]$$

- m is the reduced mass of the system

The Cross Section

- We write the cross section thus: $\sigma(E) = \frac{S(E)}{E} e^{\frac{-b}{E^{1/2}}}$

- S(E) is the "area" of the reaction - set by the deBroglie wavelength of the particle
- S(E) varies slowly with E
- Now change variable to E and rewrite <v>

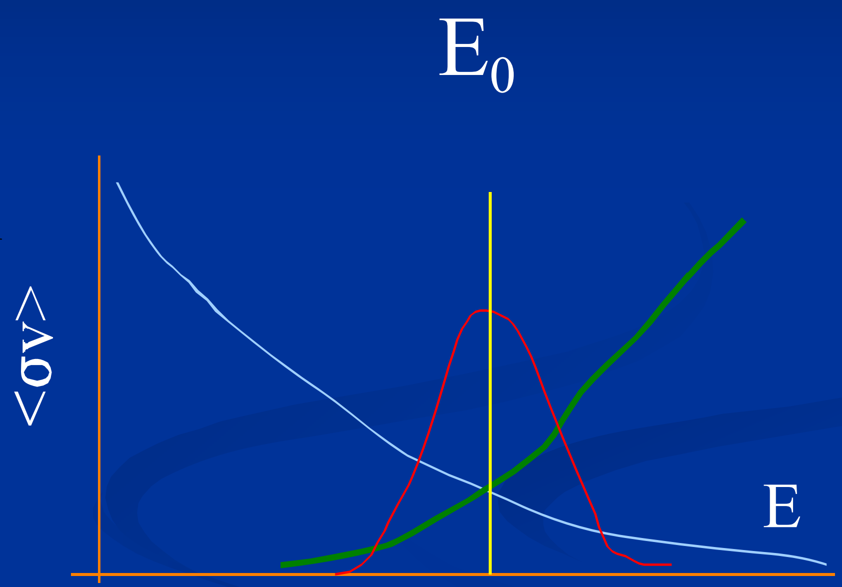
$$< \sigma v >= \int_0^\infty \sigma(E) v(E) f(E) dE$$

$$assume\ \ f(e)dE\ to\ be\ Maxwellian$$

$$< \sigma v >= \left(\frac{8}{m\pi}\right)^{\frac{1}{2}} (kT)^{-3/2} \int_0^\infty S(E) e^{\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)} dE$$

Gamow Peak

- $<\sigma v>$ is the combination of two pieces:
  - $e^{-E/kT}$ (Maxwellian) decreases with increasing E and represents the decreasing number of particles with increasing E.
  - $e^{-b/SQRT(E)}$ (Penetration) increases with E and represents the increasing probability of a reaction with E [speed].
- E_0 is the peak energy for $<\sigma V>$ and can be thought of as the most effective energy for that T



Analytically

- S(E) varies slowly with E and its primary contribution is at E_0 ==> want $S(E_0)$
- Next approximate the Peak by a Gaussian and you get:
  - See Clayton For the Approximation formulae

$$< \sigma v >= \left(\frac{8}{m\pi} \right)^{\frac{1}{2}} \frac{S(E_0)}{(kT)^{3/2}} \frac{4}{\sqrt{3}} \left(E_0 kT \right)^{\frac{1}{2}} \frac{\sqrt{\pi}}{2} e^{-\frac{3E_0}{kT}}$$

Nuclear Burning Stages and Processes

- Main Sequence Energy Generation
- pp Hydrogen Burning
  - pp cycle: $H(H,e^{+}\nu_{e})\ D\ (H,\gamma)\ He^{3}$ then $He^{3}\ (He^{3},2p)\ He^{4}$
  - Slowest Reaction is: $H(H,e^{+}\nu_{e})\ D$
    - The cross section for the above has never been measured. It also depends on the weak interaction which governs β decay : $p \rightarrow n + e^{+} + \nu_{e}$
  - For pp $T \sim 10^{7}$ K so this is the energy source for the lower main sequence (mid F - 2 $M_{\odot}$ and later)
  - $\varepsilon = \varepsilon_{0}\rho T^{4}$ for the pp cycle

Reactions of the PP Chains

Reaction	Q (MeV)	Average ν Loss (MeV)	S_0 (KeV barns)	dS/dE (barns)	B	τ (years)†
$H(p,\beta^+\nu)D$	1.442	0.263	$3.78(10^{-22})$	$4.2(10^{-24})$	33.81	$7.9(10^{9})$
$D(p,\gamma)He^3$	5.493		$2.5(10^{-4})$	$7.2(10^{-6})$	37.21	$4.4(10^{-8})$
$He^3(He^3,2p)He^4$	12.859		$5.0(10^{3})$		122.77	$2.4(10^{5})$
$He^3(\alpha,\gamma)Be^7$	1.586		$4.7(10^{-1})$	$-2.8(10^{-4})$	122.28	$9.7(10^{5})$
$Be^7(e^-,\nu)Li^7$	0.861	0.80				$3.9(10^{-1})$
$Li^7(p, \alpha)He^4$	17.347		$1.2(10^{2})$		84.73	$1.8(10^{-5})$
$Be^7(p,\gamma)B^8$	0.135		$4.0(10^{-2})$		102.65	$6.6(10^{1})$
$B^8(\beta^+\nu)Be^{8*}(\alpha)He^4$	18.074	7.2				$3(10^{-8})$

† Computed for $X = Y = 0.5$, $\varrho = 100$, $T_6 = 15$

The CNO Cycle

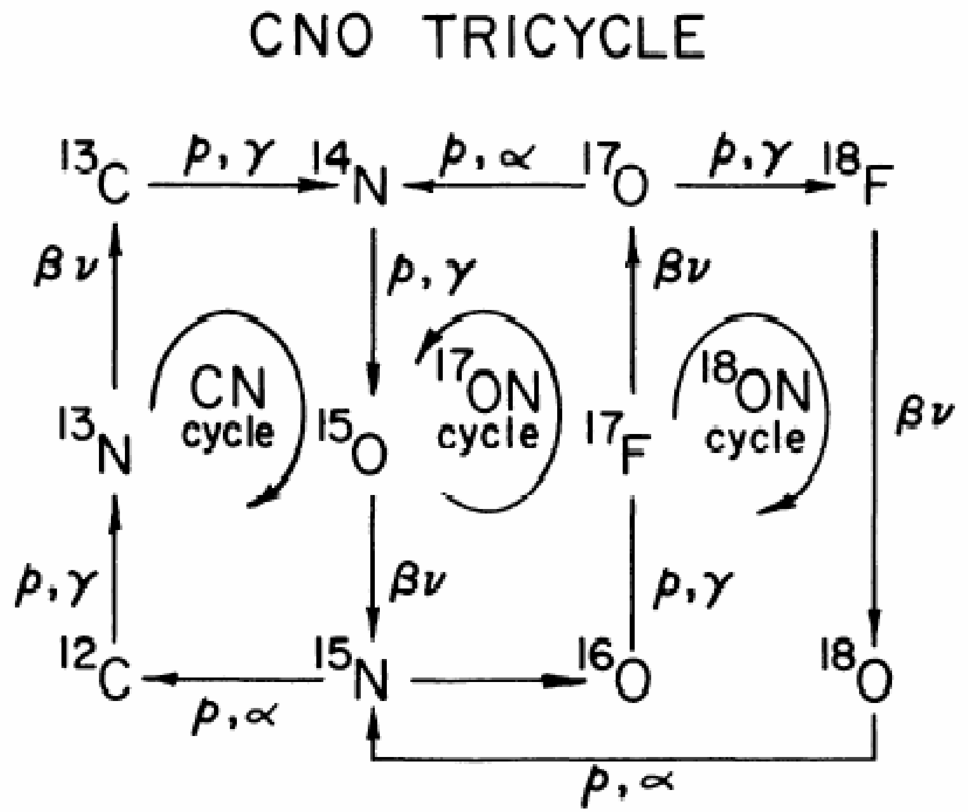


FIG. 1.—CNO Tri-cycle: Instead of ^{17}O returning to ^{14}N via $^{17}O(p, \alpha)$, there may be a competive path via $^{17}O(p, \gamma)^{18}F(e^{+}\nu)^{18}O(p, \alpha)^{15}N$.

The CNO Cycle

- Effective at $T > 2(10^7)$ K
  - Mass > 2 $M_{\odot}$ (O through early F stars)
  - The main energy generation is through the CN Cycle
  - The first two reactions are low temperature reactions. They produce ^{13}C in zones outside the main burning regions. This material is mixed to the surface on the ascent to the first red-giant branch: $^{12}C/^{13}C$ is an indicator of the depth of mixing - not a signature that the CNO process has been the main energy generator.
  - ^{14}N is produced by incomplete CN burning (and is the main source of that element).
  - The cycle when complete is catalytic.
  - $\varepsilon = \varepsilon_0 \rho T^{16}$ for the CNO cycle

CNO Reactions

Reactions of the CNO Cycle						
Reaction	Q (MeV)	Average ν Loss (MeV)	S_0 (KeV barns)	dS/dE (barns)	B	$\text{Log}(\tau\rho X_H)$ (years)†
$^{12}C(p,\gamma)^{13}N$	1.944		1.40	$4.261(10^{-3})$	136.93	2.30
$^{13}N(\beta^{+}\nu)^{13}C$	2.221	0.710				
$^{13}C(p,\gamma)^{14}N$	7.550		5.50	$1.34(10^{-2})$	137.20	1.70
$^{14}N(p,\gamma)^{15}O$	7.293		2.75		152.31	4.21
$^{15}O(\beta^{+},\nu)^{15}N$	2.761	1.00				
$^{15}N(p,\alpha)^{12}C$	4.965		$5.34(10^{4})$	$8.22(10^{2})$	152.54	-0.21
$^{15}N(p,\gamma)^{16}O$	12.126		27.4	$1.86(10^{1})$	152.54	
$^{16}O(p,\gamma)^{17}F$	0.601		10.3	$-2.81(10^{-2})$	166.96	5.85
$^{17}F(\beta^{+}\nu)^{17}O$	2.762	0.94				
$^{17}O(p,\alpha)^{14}N$	1.193	Resonant Reaction			167.15	3.10

† Computed for $T_6 = 25$

Pre-Main Sequence Nuclear Sources

- $^{6}Li\ (H,{}^{4}He)\ {}^{3}He$
- $^{7}Li\ (H,{}^{4}He)\ {}^{4}He$
- $^{10}B\ ({}^{2}H,{}^{4}He)\ 2{}^{4}He$
- $^{9}Be\ (2H,2{}^{4}He)\ {}^{3}He$
- $D\ (H,\gamma)\ {}^{3}He$

- These are parts of the pp cycle and destroy any of these elements that are present. But these species still exist! How do they survive?

Post-Main Sequence Processes

The Triple α Process

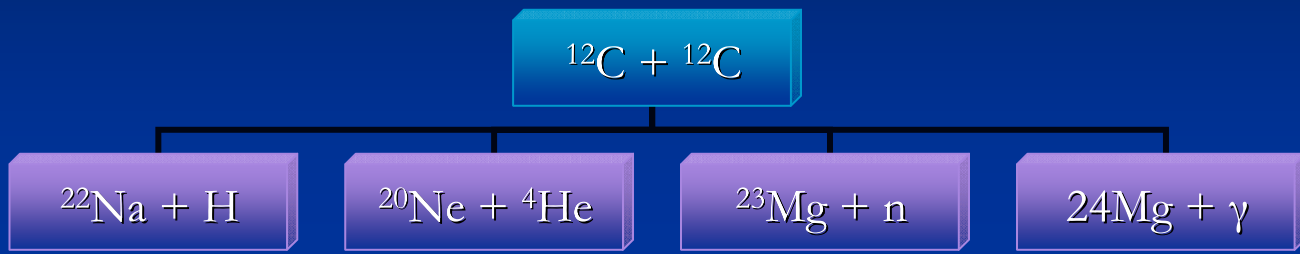
- Mass Criteria: $M/M_{\odot} > 0.5$
- $T_c \approx 10^8$ K
- Summary: $3\ {}^4He \rightarrow {}^{12}C + \gamma$ $\quad Q = 7.27$ MeV
- Details: ${}^4He\ ({}^4He,\ )\ {}^8Be\ ({}^4He,)\ {}^{12}C$
  - ${}^4He\ ({}^4He,\ )\ {}^8Be$ is endothermic
  - The third ${}^4He$ must be present when ${}^4He\ ({}^4He,\ )\ {}^8Be$ occurs or $Be^8 \rightarrow 2He^4$ occurs (Timescale is $2.6(10^{-16})$ s)
  - This is a three body collision! It happens because ${}^{12}C$ has an energy level at the combined energy of ${}^8Be + {}^4He$. This is a resonance reaction.
  - $\varepsilon = \varepsilon_0 \rho^2 T^{30}$ for 3α

Other Common Reactions

These happen along with 3α

- A Very Important Reaction: $^{12}C(^{4}He,)^{16}O$
  - This reaction has a very high rate at the temperatures and densities of 3α
  - After 3α one has C and O.
- $^{14}N\ (^{4}He,e^{+}\nu_{e})\ ^{18}O\ (^{4}He,)\ ^{22}Ne$
  - This burns to completion under 3α conditions so all N is destroyed.

Carbon Burning



- C is the next fuel - it is the lowest Z nuclei left after 3α
  - $T_c > 6(10^8)$ K
  - $\varepsilon = \varepsilon_0 \rho T^{32}$ for carbon burning

Oxygen Burning

$^{16}O + ^{16}O$

- $^{32}S + \gamma$
- $^{31}P + p$
- $^{31}S + n$
- $^{28}Si + \alpha$
- $^{24}Mg + 2\alpha$

Neutrino Energy Loss

- Cross Section is $\sigma \sim 10^{-44}$ cm^2
- Interaction Probability $\simeq \sigma n_b R$
  - n_b = baryon number
  - R = radius of star
- Neutrinos are a source of energy loss but do not contribute to the pressure

Davis Experiment

- ${}_{17}Cl^{37} + \nu_e \rightarrow {}_{18}Ar^{37} + e^-$
- The threshold energy is 0.81 MeV for the reaction.
- This is a "superallowed" transition as the quantum numbers of excited ${}_{18}Ar^{37}$ at 5.1 MeV are the same as the ground state in ${}_{17}Cl^{37}$.
  - We would expect to be able to detect a single event at $E > 5.1$ MeV
- Primary ν_e sources in the Sun are the pp and CN cycles.

Sources of Solar Neutinos

Reaction	Neutrino Energy		Relative Importance
	Mean	Max	
$p + p \rightarrow D + e^+ + \nu_e$	0.26	0.42	6.0
$p + p + e^- \rightarrow D + \nu_e$	1.44	1.44	$1.55(10^{-2})$
$Be^7 + e^- \rightarrow Li^7 + \nu_e$	0.86	0.86	0.4
$B^8 \rightarrow Be^{8*} + e^+ + \nu_e$	7.2	14.0	$4.5(10^{-4})$
$N^{13} \rightarrow C^{13} + e^+ + \nu_e$	0.71	1.19	$3.7(10^{-2})$
$O^{15} \rightarrow N^{15} + e^+ + \nu_e$	1.00	1.70	$2.6(10^{-2})$

Experimental Results

- The only neutrino the Davis experiment can see is the B^8 neutrino - the least significant neutrino source in the chains.
- Original Predicted Rate was 7.8 SNU with a detected rate of 2.1 ± 0.3 SNU
- GALLEX tests the p + p neutrino production: its rate is about 65% of the predicted values.
  - Better solar models are cutting the predicted rates
    - Diffusion
    - Convection
    - Better opacities