Continuous Opacity Sources

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- Principal Sources:
  - Bound-Bound Transitions
  - Bound-Free
  - Free-Free (Bremstralung)
  - Electron Scattering (Thompson & Compton)
  - Molecular Transitions
- We consider only H or H-Like cases
  - We will do this classically (and correct to the QM result)
  - Stars are mostly H (in one form or another).

Dominant Opacity Sources

Type of Star	Species	Type	
Cool Stars (G-M) of Normal Composition	H^-	b-f	Dominant
Warmer A-F stars	H^-	f-f	Secondary
	H	b-f	Dominant
	H	f-f	Secondary
Interiors and hottest stars	H	f-f	

Caveats and Details

- At High Temperatures: $(1-e^{-h\nu/kT}) \rightarrow 0$ so all of the bb, bf, and ff sources go to 0!
- Electron scattering takes over (is always there and may be important).
  – Free-Free is not the same as electron scattering: Conservation of momentum says a photon cannot be absorbed by a free particle!
- In principal we start with a QM description of the photon - electron interaction which yields the cross section for absorption/scattering of the photon of energy $h\nu$, call the cross section $a_i(\nu)$.

The Opacity Is

- The opacity (g/cm^2) for the process is

$$K_i(\nu) = n_i a_i(\nu)/\rho$$

- n_i is the number density (#/cm^3) of the operant particles
- The subscript i denotes a process/opacity
- The total opacity is
- $$K_{total} = \sum K_i(\nu)$$

Compton Scattering

- This process is important only for high energy photons as the maximum change is 0.024Å.
- Reference: Eisberg -- Fundamentals of Modern Physics p. 81 ff.

Electron Scattering Conditions

- Collision of an electron and a photon
  - Energy and Momentum **Must** be conserved
- In stellar atmospheres during photon-electron collisions the wavelength of the photon is increased (assume the $E_{electron} <$ Photon ($h\nu$))
  - At 4000Å: $h\nu = 4.966(10^{-12})$ ergs
  - $\frac{1}{2} mv^2 < 5(10^{-12})$ ergs ==> $v < 10^8$ cm/s
  - At 5000K $v_{RMS} = 6.7(10^5)$ cm/s
  - At 100000K $v_{RMS} = 3(10^6)$ cm/s

Thompson Scattering

Classical Electron Scattering

- Reference: Marion - Classical Electromagnetic Radiation p. 272 ff
- Low Energy Process: v << c
- Energy Absorbed from the EM field is

$$\frac{dE}{dt} = \frac{2}{3}\frac{e^2}{c^3}a^2 \qquad (\#)$$

- a = acceleration of the electron: $a = eE/m_e$ and E is the magnitude of the electric field.

Thompson Scattering

- Field Energy Density = $<E^2/4\pi>$ (Time Average)
- Energy Flux Per Electron = c $<E^2/4\pi>$
- Now Take the Time Average of (#):
- But that has to be the energy flux per electron times the cross section which is σ_T c $<E^2/4\pi>$.

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{2}{3}\frac{e^2}{c^3} < a^2 >$$

$$= \frac{2}{3}\frac{e^2}{c^3}\frac{e^2}{m_e^2}\left\langle E^2 \right\rangle$$

The Thompson Cross Section

$$\sigma_T c \left\langle \frac{E^2}{4\pi} \right\rangle = \frac{2}{3} \frac{e^2}{c^3} \frac{e^2}{m_e^2} \left\langle E^2 \right\rangle$$

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65(10^{-25})\ cm^2$$

At High Temperatures this breaks down: $T \geq 10^9$ K

Electron Scattering Opacity

- $K_e = \sigma_T N_e / \rho$
- ***There is no frequency dependence!***
- Scattering off other ions is unimportant:
- Cross Section Goes as $(1/m)^2$ so for ions $(1/Am_H)^2$ while for electrons it goes as $(1/m_e)^2$
- Ratio: Ions/Electrons = $(m_e/Am_H)^2 = (1/A(1840))^2 < 10^{-6}$

Rayleigh Scattering: σ_R

- Scattering of a low energy photon by a bound electron.
- Classically: Rayleigh scattering occurs when a photon of energy less than the atomic energy spacing is absorbed.
  - The electron then oscillates about the unperturbed energy level (harmonically).
  - The electron reradiates the same photon but remains in the same energy state.

The Rayleigh Cross-Section

- The cross section is: $\sigma = \sigma_T/(1-(\nu_0/\nu)^2)^2$
  - Where $h\nu$ is the photon energy
  - $h\nu_0$ is the restoring force for the oscillator
- When $\nu << \nu_0$: $\sigma_R = \sigma_T (\lambda_0/\lambda)^4$
  - Now since $\nu << \nu_0$ we have ❍E << kT
  - Which implies T ~ 1000 K for this process.

Free-Free Opacities

Absorption Events

- Bremstralung
  - Electron moving in the field of an ion of charge Ze emits or absorbs a photon:
    - Acceleration in field produces a photon of hν
    - De-acceleration in field consumes a photon of hν
- Consider the Emission Process
  - Initial Electron Velocity: v′
  - Final Electron Velocity: v
- Conservation of Energy Yields
- $\frac{1}{2} m_e v^2 + h\nu = \frac{1}{2} m_e v'^2$

Energy Considerations

- Energy Absorbed: $dE/dt = 2/3\ (e^2/c^3)\ a^2$ where $a = eE/m_e$ (a is the acceleration)

$$E = \int_{-\infty}^{\infty} \frac{dE}{dt} dt = \frac{2}{3} \frac{e^2}{c^3} \int_{-\infty}^{\infty} a^2 dt$$

- Most energy is absorbed during the time $t \approx b / v'$ when the electron is close to the ion
  - b is called the impact parameter
  - b is the distance of closest approach
- Acceleration is $\sim Ze^2/m_e b^2$
- $E_{abs} \approx 2/3\ (e^2/c^3)\ (Ze^2/m_e b^2)^2\ (b / v')$
- $\approx 2/3\ ((Z^2 e^6)/(m_e^2\ c^3\ b^3\ v'))$

Frequency Dependence

Expand dE/dT in a Fourier Series

$$\frac{dE(\nu)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dE(t)}{dt}e^{-2\pi i\nu t}dt$$

- Greatest contribution is when 2πνt 🕘 1 during time t 🕘 b / v′
  - Therefore 2πν b / v′ = 1 or 2πν = v′ / b
- The energy emitted per electron per ion in the frequency range d is
  - $dq_\nu = 2\pi b\ db\ E_{abs}$ Why? 2πb db is an area!
  - dq_ν = energy emitted per electron per ion per unit frequency

Bremstralung Energy

- Total Energy Emitted: $n_i n_e$ $v' f(v') dv'$ dq_ν
  - – n_i = Ion number density
  - – n_e = electron number density (note that the electron flux is n_e $v' f(v') dv'$)
- The reverse process defines the Bremstralung absorption coefficient a_ν giving the absorption per ion per electron of velocity v from the radiation field.
- In TE: Photon Energy Density = $U_{\nu p}$ = $(4\pi/c)$ $B_\nu(T)$

The Absorption Coefficient

- Net energy absorbed must be the product of the photon flux (photon energy = hν) $cU_{\nu p}d\nu$ and $n_i n_e f(v)dva_\nu$ and $(1\text{-}e^{-h\nu/kT})$ or
  - $cU_{\nu p}d\nu\ n_i n_e f(v)dva_\nu\ (1\text{-}e^{-h\nu/kT})$
- But in TE that must be equal to the emission:
  - $cU_{\nu p}d\nu\ n_i n_e f(v)dva_\nu\ (1\text{-}e^{-h\nu/kT}) = n_i n_e\ v'f(v')dv'\ dq_\nu$
- $a_\nu = \pi/3\ (Z^2e^6)/(hc\ m_e^{\,2}\ \nu^3\ v)$
- This is off by $4/\sqrt{3}$ from the exact classical result.

The Bremstralung Opacity

$$\kappa_{ff}(\nu) = \int_0^\infty n_i n_e f(V) a_\nu g_{ff}(\nu, V) dV$$

- V is v in the previous equations
- This reduces to

  $\kappa_{ff}(\nu) = 4/3\ n_i n_e\ (2\pi/3m_e kT)^{1/2}\ ((Z^2 e^6)/(hc\ m_e \nu^3))\ g_{ff}(\nu)$
- $g_{ff}(\nu)$ is the mean Gaunt factor and the result has been corrected to the exact classical result.

Bound Free Opacities

Transition from a Bound State to Continuum or Visa Versa

- This process differs from the free-free case due to the discrete nature of one of the states
- N^{th} Discrete State:

$$E_n = \frac{-m_e Z^2 e^4}{2\hbar^2 n^2} \equiv -\frac{I_H Z^2}{n^2}$$

- Then the electron capture/ionization process must satisfy:
- $\frac{1}{2} m_e v^2 - E_n = h\nu$

Bound Free

- V is the velocity of the ejected or absorbed electron.
- Semiclassical treatment of electron capture
  - Electron Initial Energy: ½ $m_e v^2$ and is positive
  - The energy decreases in the electric field as it accelerates seeing ion of charge Ze
  - Q: Why does it loose energy as it accelerates?
  - A: It radiates it away
- The energy loss per captured electron may be estimated as:
  - $dq_\nu = 2\pi b db E_{abs} = (8\pi^2/3)\,((Z^2e^6)/(m_e^2\, c^3\, v'^2 d\nu))$

Cross Section

- The cross section for emission of photons into frequency interval $d\nu$ is defined by
  - $h\nu\, d\sigma_\nu = dq_\nu = h\nu\, (d\sigma_\nu/d\nu)\, d\nu$
- The final state is discrete so define σ_{cn} as the cross section for capture into state n (energy E_n) characterized by n in the range (n, n+dn) so that $d\sigma_\nu = \sigma_{cn} dn$. Then
  - $h\nu\, (d\sigma_\nu/d\nu) = h\nu\, \sigma_{cn}\, dn/d\nu$
  - Solve for σ_{cn} and use $E_n = -I_H Z^2/n^2$ to get $dn/d\nu$
- Thus: $\sigma_{cn} = ((2I_H Z^2/n^2)/(h^2 \nu n^3))(dq_\nu/d\nu)$

  $= (32/3)\, \pi^4\, ((Z^2 e^{10})/(m_e c^3 h^4 v^2 \nu n^3))$

Photoionization

The reverse process is related by detailed balance

- Let $\sigma_{\nu n}$ = photoionization cross section
- The number of photons absorbed of energy $h\nu$ with the emission of electrons of energy $\frac{1}{2}m_e v^2 - E_n$ from the n^{th} atomic state is
  - $(cU_{\nu p}/h\nu)\, d\nu\, \sigma_{\nu n}\, N_n\, (1-e^{-h\nu/kT})$
  - N_n is the number of atoms in state n
- The reverse process: the number of electrons with initial velocities v captured per second into state n is $N_i\, \sigma_{cn}\, n_e\, v\, f(v)\, dv$

State Population

- The Boltzmann Equation for the system is:

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{\frac{-I_H Z^2 (1-\frac{1}{n^2})}{kT}}$$

- where $g_1 = 2$ and $g_n = 2n^2$ (H-like ions) and the Saha equation is

$$\frac{N_i N_e}{N} = 2\frac{g_i}{U_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{\frac{-I_H Z^2}{kT}}$$

- Detailed Balance says:
- $(cU_{\nu p}/h\nu)d\nu\sigma_{\nu n}N_n(1-e^{-h\nu/kT}) = N_i\sigma_{cn}n_e v f(v)dv$

The Bound Free Coefficient

- We assume
  - Maxwellian distribution of speeds
  - Boltzmann and Saha Equations
- $\sigma_{\nu n} = (g_i/g_n)\,(m_e v c/h\nu)^2\,\sigma_{cn}$
- We assume the Z-1 electrons in the atom do not participate, set $g_i = 1$, and correct for QM then

$$\sigma_{\nu n} = \frac{64\pi^4}{3\sqrt{3}}\frac{m_e Z^4 e^{10}}{h^6 c \nu^3 n^5} g_{bf}(\nu, n)$$

Limits and Conditions

- Photon *must* have $h\nu > E_n$
  - $\sigma = 0$ for $h\nu < En\ (= I_H Z^2/n^2)$
- Recombination
  - Levels coupled to ν by bound free processes have
    - E_n for $n > n^*$ where $n^* = (I_H Z^2/h\nu)^{1/2}$
- Total Bound Free Opacity:

$$\kappa_{bf}(\nu) = \sum_n \frac{N_n}{\rho}\sigma_{\nu n}$$

Other Opacities

Source	BF	FF	Other
H I	Yes	Yes	Rayleigh
H2-	Yes	Yes	
H-	Yes	Yes	
He I	Yes	Yes	Rayleigh
He II	Yes	Yes	
He-		Yes	
Low Temperature Atomic: T < 10000K			
C I	Yes	Yes	
Mg I	Yes	Yes	
Si I	Yes	Yes	
Al I	Yes	Yes	
Intermediate Atomic: 10000 < T < 20000			
Mg II	Yes	Yes	
Si II	Yes	Yes	
Ca II	Yes	Yes	
N I	Yes	Yes	
O I	Yes	Yes	
Hot Atomic: T > 20000K			
C II – IV	Yes	Yes	
N II – V	Yes	Yes	
Ne I - VI	Yes	Yes	

The Rosseland Mean Opacity

$$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

More Spectroscopic Notation

H Like Only

- Electron and proton have
  - Spin ½
  - Angular momentum $\hbar/2$ and each has a corresponding magnetic moment
- The magnetic moment of the electron interacts with both the orbital magnetic moment of the atom (spin-orbit) and the spin of the proton (spin-spin).
  - Spin-orbit: fine structure (multiplets)
  - Spin-spin: hyperfine structure

Notation

- Principal Quantum Number n
- Orbital Quantum Number ℓ: $\ell \leq n - 1$
  - S P D F
  - ℓ = 0 1 2 3
- Total angular momentum of a state specified by $\ell = \ell[(\ell+1)]^{1/2} \hbar$
- Spin angular momentum (s=1/2): $[s(s+1)]^{1/2} \hbar = \sqrt{3}\, \hbar /2$
- Orbital and spin angular momentum interacts to produce a total angular momentum quantum number j $= \ell \pm 1/2$: S States have j = 1/2, P states have j = 1/2, 3/2; and D has j = 3/2, 5/2

H Like Notation

$$n^{2s+1}\ell_j$$