Radiative Equilibrium

The Condition of Radiative Equilibrium

- Modes of Energy Transport
  - Radiative
  - Convective
  - Conduction - Not important except in WD's etc
- When all energy is transported by radiation the condition of radiative equilibrium applies.
- Schwarzschild: $\tau < 1$: Radiative

  $\tau > 1$: Convective
- Convection is driven by Hydrogen ionization.

Radiative Equilibrium

- The Energy Removed from the Beam:

$$\int_0^\infty d\nu \oint \mathrm{K}_\nu I_\nu \, d\omega = 4\pi \int_0^\infty \mathrm{K}_\nu J_\nu \, d\nu$$

- K_ν - total absorption coefficient
- Total Energy Replaced In Beam:

$$\oint d\omega \int_0^\infty j_\nu \, d\nu = 4\pi \int_0^\infty \mathrm{K}_\nu S_\nu \, d\nu$$

- Radiative Equilibrium Demands:

$$\int_0^\infty \mathrm{K}_\nu J_\nu \, d\nu = \int_0^\infty \mathrm{K}_\nu S_\nu \, d\nu \qquad (*)$$

Flux Constancy

- Take the Equation of Transfer and Integrate over all Frequencies and Solid Angles

$$\frac{1}{\rho}\frac{d}{dz}\left(\int_0^\infty \oint I_\nu \mu d\omega\right) = \oint d\omega \int_0^\infty j_\nu d\nu - \int_0^\infty d\nu \oint \mathrm{K}_\nu I_\nu d\omega$$

- But Under Radiative Equilibrium the Right Side = 0!

$$\frac{1}{\rho}\frac{d}{dz}\left(\int_0^\infty \oint I_\nu \mu d\omega\right) = 0$$

$$\frac{dF}{dz} = 0$$

- Therefore the Flux is constant with depth!
- Total Flux uniquely specified for each atmosphere

Consider A Blackbody

- A Small Opening in A TE Cavity

$$F_{BB}(\nu) = 2\pi \int_{-1}^{1} I_\nu(\mu)\mu d\mu$$

$$= 2\pi B_\nu(T) \int_{0}^{1} \mu d\mu$$

$$= \pi B_\nu(T)$$

- Total Flux is the Integral over Frequency:

$$\int_0^\infty F_{BB}(\nu) d\nu = \pi \int_0^\infty B_\nu(T) d\nu = \sigma T^4$$

- Define Teff:

$$\frac{\sigma T^4}{\pi} = F = \int_0^\infty F_\nu d\nu$$

One Last Consideration

Look at (*) Again

$$\int_0^\infty \mathrm{K}_\nu J_\nu \, d\nu = \int_0^\infty \kappa_\nu J_\nu \, d\nu + \int_0^\infty \sigma_\nu J_\nu \, d\nu$$

$$= \int_0^\infty \mathrm{K}_\nu S_\nu \, d\nu$$

$$= \int_0^\infty \kappa_\nu B_\nu \, d\nu + \int_0^\infty \sigma_\nu J_\nu \, d\nu$$

$$\therefore$$

$$\int_0^\infty \kappa_\nu J_\nu \, d\nu = \int_0^\infty \kappa_\nu B_\nu \, d\nu$$

Scattering has cancelled out as it puts as much energy into the beam as it takes out!

HR Diagram

Chapter 13 – "Stellar Rotation"
Böhm-Vitense – *Introduction to Stellar Astrophysics* Vol 1

Landolt-Börnstein