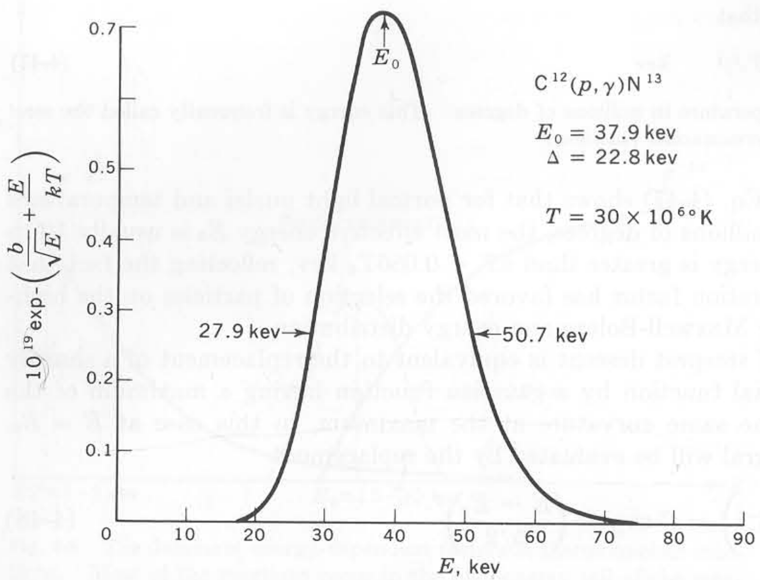

Fig. 4-7 The Gamow peak for the reaction $C^{12}(p,\gamma)N^{13}$ at $T = 30 \times 10^6$ °K. The curve is actually somewhat asymmetric about E_0, but it is nonetheless adequately approximated by a gaussian.

The most effective particles have energies ranging only about 10 kev from the most effective energy E_0. This range of energies is quite small compared to the average energy separation of quasistationary nuclear states in the light nuclei and accounts for the fact that the effects of nuclear forces, which are lumped into $S(E)$, may often be considered to be constant. The factor $S(E)$ will generally change by a large percentage of its value over the range Δ only if there is a nuclear resonance near the range of effective stellar energies, but in that case the resonant reaction rates must be employed. These will be discussed later.

Suffice it to say, then, that the experimentally measured cross-section factor can be plotted, as in Fig. 4-5 for the $C^{12}(p,\gamma)N^{13}$ reaction, and extrapolated to the range of stellar energies. This extrapolation can be made with considerably greater accuracy than could the extrapolation of the cross-section itself. In fact the solid line of Fig. 4-5 is a semitheoretical fit to the points, made in a manner to be explained later. From this analysis one can describe $S(E)$ at low energies by its intercept and slope. In the particular case of the $C^{12}(p,\gamma)N^{13}$ reaction, for instance, one finds that $S(E = 0) = 1.20$ kev barns and $dS/dE = 5.81 \times 10^{-3}$ barn. In like manner, the cross-section factor has been determined with varying degrees of accuracy for most of the important energy-generating reactions in stellar interiors.

Making the approximate substitution

$$\exp\left(-\frac{E}{kT} - bE^{-\frac{1}{2}}\right) \approx e^{-\tau} \exp - \left(\frac{E - E_0}{\Delta/2}\right)^2 \tag{4-53}$$

where the quantity $3E_0/kT$ has been designated by τ, the reaction rate per pair of particles may be written from Eq. (4-44) as

$$\lambda = \left(\frac{8}{\mu\pi}\right)^{\frac{1}{2}} \left(\frac{1}{kT}\right)^{\frac{3}{2}} e^{-\tau} \int_0^\infty S(E) \exp\left[-\left(\frac{E - E_0}{\Delta/2}\right)^2\right] dE \tag{4-54}$$

Once again it is obvious that if $S(E)$ is nearly constant, most of the value of the integral comes from values of E near E_0. The first approximation for well-behaved cross-section factors is to treat $S(E)$ as a constant S_0 defined as the value of $S(E_0)$. It is also evident that negligible error will be committed by extending the lower limit of the integral to minus infinity.[1]

Problem 4-12: Show that when S_0 is expressed in cgs units of erg cm² and when the approximations indicated above are performed, one obtains for the reaction rate per pair

$$\lambda = \frac{4.50 \times 10^{14}}{AZ_1Z_2} S_0\tau^2 e^{-\tau} \quad \text{cm}^3/\text{sec} \tag{4-55}$$

Since kev is a more appropriate energy unit than ergs, and barns a more appropriate cross section than cm², it is more common to express S_0 in units of kev barns, e.g., Fig. 4-5. We shall follow that practice throughout this book. The reaction rate per pair then becomes numerically

$$\lambda = \frac{7.20 \times 10^{-19}}{AZ_1Z_2} S_0 \text{ (kev barns)} \tau^2 e^{-\tau} \quad \text{cm}^3/\text{sec} \tag{4-56}$$

whereas the reaction rate is obtained by multiplying by the number of pairs per unit volume:

$$r_{12} = (1 + \delta_{12})^{-1} N_1 N_2 \lambda_{12} \tag{4-57}$$

The convenience of writing the reaction rate in this form is that the all-important temperature dependence of the rate is entirely contained in the parameter τ.

Problem 4-13: Show that

$$\tau = 42.48 \left(\frac{Z_1^2 Z_2^2 A}{T_6}\right)^{\frac{1}{3}} \tag{4-58}$$

For any given reaction, τ is proportional to $T^{-\frac{1}{3}}$; thus one can write

$$\tau = BT_6^{-\frac{1}{3}} \tag{4-59}$$

[1] See, for instance, a table of the normal probability integral for a characteristic value of $2E_0/\Delta$.