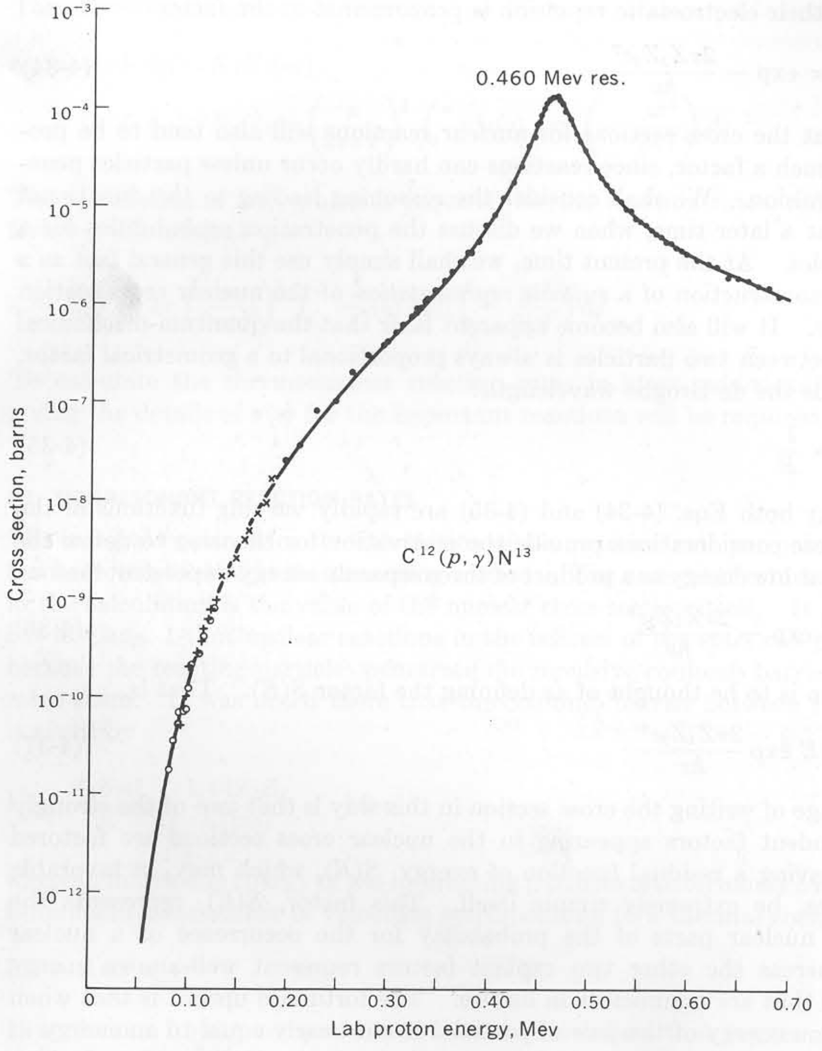

Fig. 4-4 The measured cross section for the reaction $C^{12}(p,\gamma)N^{13}$ as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to $E_p = 0.025$ Mev, which is an interesting energy for this reaction in astrophysics, appears treacherous. (*Courtesy of W. A. Fowler and J. L. Vogl.*)

unit of cross section is used in Fig. 4-4. The energy abscissa is seen to be the laboratory proton energy, $\frac{1}{2}m_pv^2$, whereas the energy to be used in Eq. (4-37) is the energy of the pair of particles in the center of mass, which is

$$E = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}v^2 = \tfrac{1}{2}\mu v^2 \tag{4-38}$$

It is evident that the energy of a pair of particles in their center of mass is related to the laboratory energy of particle 1 by the relationship

$$E = \frac{m_2}{m_1+m_2}\,E_{1,\mathrm{lab}} \tag{4-39}$$

Several interesting features are immediately obvious from Fig. 4-4. The cross section has a maximum of about 10^{-4} barn near the energy $E_{\mathrm{lab}} = 460$ kev and falls by seven orders of magnitude as the energy falls from 500 to 100 kev. It is further apparent that near 100 kev, the cross section is changing by about one order of magnitude per 25 kev. In other words, the nuclear cross sections for the interactions of charged particles vary extremely rapidly with energy at low energies. The maximum in this cross section at 460 kev is due to a resonance in the compound N^{13} system. Such resonances will be discussed later.

The point to be made at this time is that the rapidly falling cross section at low energies is due almost entirely to the effects of the exponential factor in the cross section. This exponential, sometimes called the *Gamow velocity factor*, is proportional to the probability of penetration through the coulomb repulsion. Quantitative definitions of the penetration factors will be described later. As factual evidence for the foregoing statements, the nuclear cross-section factor $S(E)$, as defined in Eq. (4-37), is shown in Fig. 4-5, along with the experimental data, which are plotted as points. The interesting fact is that the cross-section factor $S(E)$ is seen to be almost independent of energy, changing by less than a factor of 2 between 0 and 100 kev. Whereas the cross section itself changed by an order of magnitude in 25 kev near 100 kev, the cross-section factor changes by not more than 10 percent or so in 25 kev near 100 kev. These facts corroborate the statement that the cross-section factor is quite often a slowly varying function of energy that can be represented over a limited energy range as either a constant or a slowly increasing linear function of energy. We shall return to these two instructive figures often in the material that follows.

Problem 4-7: Show that if the cross section is written

$$\sigma(E) = \frac{S(E)}{E}\exp - bE^{-\frac{1}{2}} \tag{4-40}$$

the value of the parameter b is

$$b = 31.28Z_1Z_2A^{\frac{1}{2}} \qquad \mathrm{kev}^{\frac{1}{2}} \tag{4-41}$$

where A is the reduced atomic weight, defined to be

$$A = \frac{A_1A_2}{A_1+A_2} = \frac{\mu}{M_u} \tag{4-42}$$

and M_u is, as before, the mass of 1 amu. That is, if the center-of-mass energy E is expressed in units of kev, Eq. (4-41) may be used for numerical convenience. It is conventional to use these energy units in preference to cgs units in nuclear astrophysics.