Measuring stellar mass
(and radius
and temperature)

Not only a basic quantity for understanding structure & evolution of stars, but useful for planet searches.

Visual binaries — can see both stars
(nearby!)

Position vector $\underline{R}$: mass weighted average of position

$$\underline{R} = \frac{m_1 \underline{r}_1' + m_2 \underline{r}_2'}{m_1 + m_2}$$

This is zero in center of mass frame

$$\frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2} = 0$$

So $\underline{r}_1 = \frac{-m_2}{m_1+m_2}\underline{r}$ & $\underline{r}_2 = \frac{m_1}{m_1+m_2}\underline{r}$

if $\underline{r} = \underline{r}_2 - \underline{r}_1$

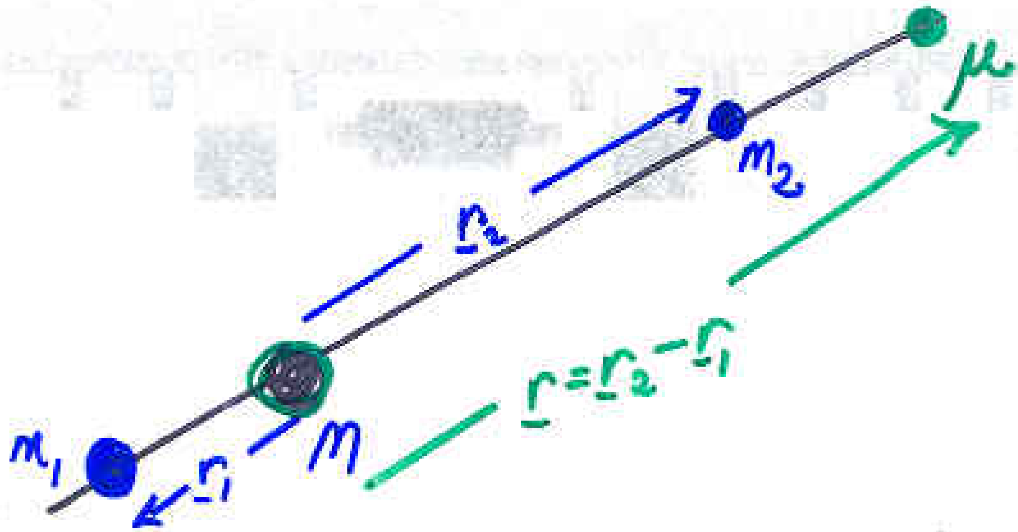
same distance to both stars so if α_1 is angular distance from c. of m. to star 1, etc

$$\frac{r_2}{r_1} = \frac{m_1}{m_2} \quad \& \quad \boxed{\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}}$$

We can derive the ratio of masses from the distances of the stars from their center of mass.

Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Two-body problem reduces to mass μ orbiting mass $M = m_1 + m_2$



Semi-major axis a of μ's orbit about M is sum of semi-major axes a_1 & a_2 of m_1 & m_2.

Kepler's 3rd law becomes:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

We have m_1/m_2 & m_1+m_2 so can derive each mass separately

But we need a expressed as a linear distance, not an angle so need to know the distance to the star

$\boxed{Q}$ What does a typical parallax distance error do to the mass estimate?

→ There are few visual binaries known and few good parallax estimates, so good estimates of stellar mass are rare.

Eclipsing binaries

If binary system is far enough away that the components can't be separated visually, the spectrum can still give information about the velocities of each component.

For stars too distant to be resolved as visual binaries, we can use velocity information if spectral lines are visible from both stars.

For circular orbits velocity is constant in magnitude: (not direction!)

$$v = \frac{2\pi a}{P}$$

So ratio of masses can be found from velocities:

$$\frac{m_2}{m_1} = \frac{v_1}{v_2}$$

If binary system is not exactly edge-on, what would velocity curve look like?

What if orbits were elliptical?

In general we don't know inclination of system, which is a problem

BUT

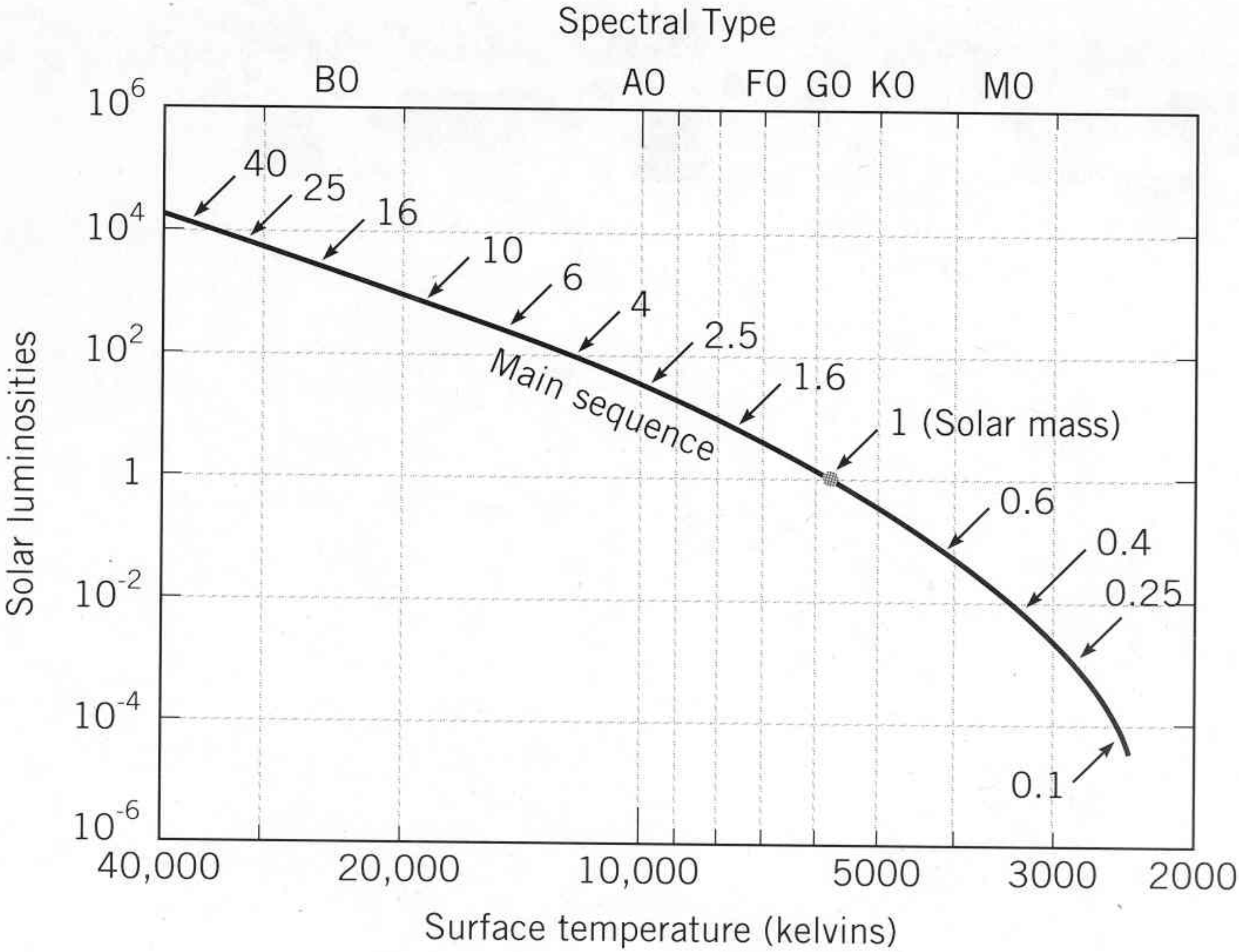
if the binary is an eclipsing system we know it must be very close to edge-on.

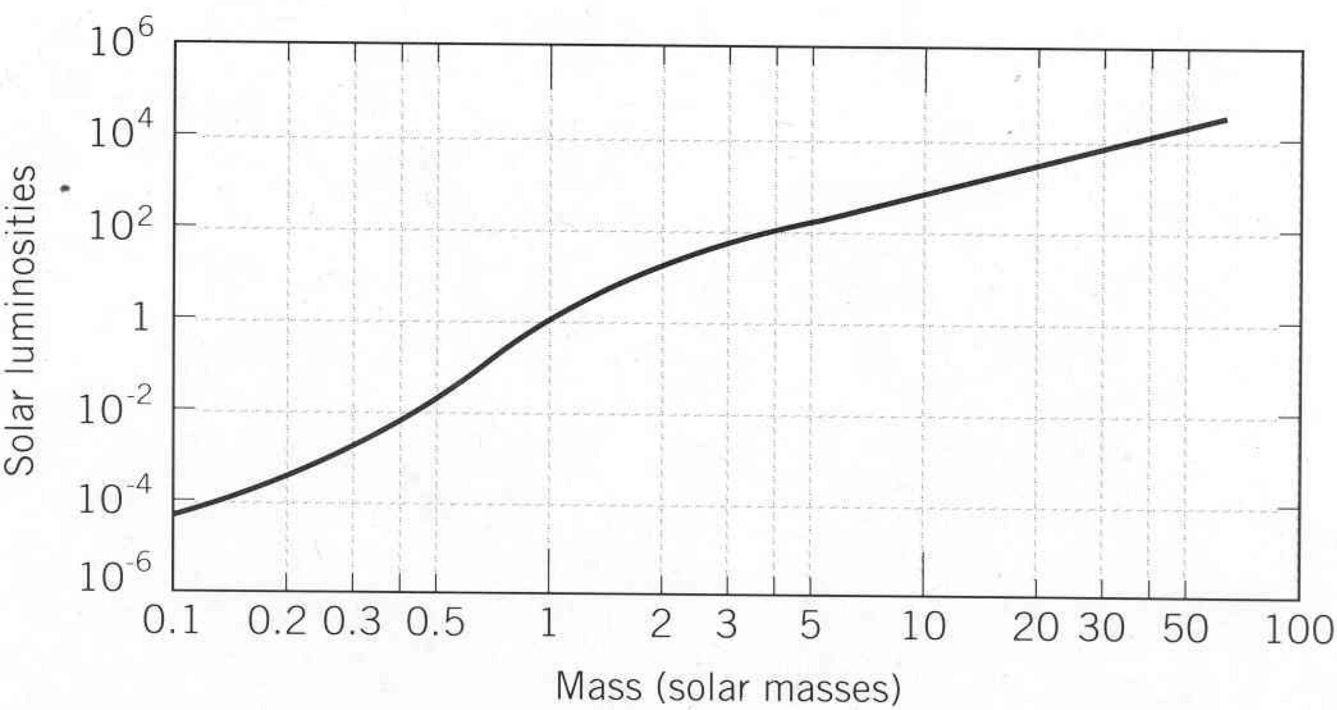
Can then use reduced-mass frame & Kepler's 3rd law:

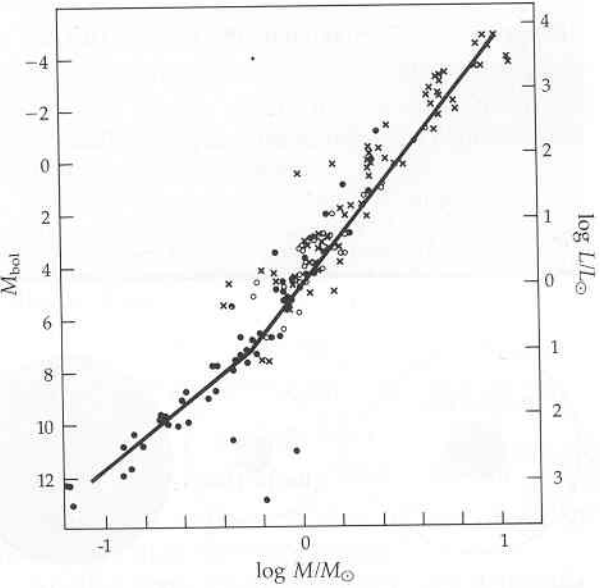
$$m_1 + m_2 = \frac{P}{2\pi G} (v_1 + v_2)^3$$

Can also measure radii of stars from accurate light curves

$$\longrightarrow T_{eff} \quad \text{since } F_{surf} = \sigma T_{eff}^4$$


Spectral Type
B0
A0
F0
G0
K0
M0
10^6
10^4
10^2
1
10^{-2}
10^{-4}
10^{-6}
Solar luminosities
40
25
16
10
6
4
2.5
1.6
1 (Solar mass)
0.6
0.4
0.25
0.1
Main sequence
40,000
20,000
10,000
5000
3000
2000
Surface temperature (kelvins)


10^6
10^4
10^2
1
10^{-2}
10^{-4}
10^{-6}
Solar luminosities
0.1
0.2
0.3
0.5
1
2
3
5
10
20
30
50
100
Mass (solar masses)



(B)

Stellar sizes

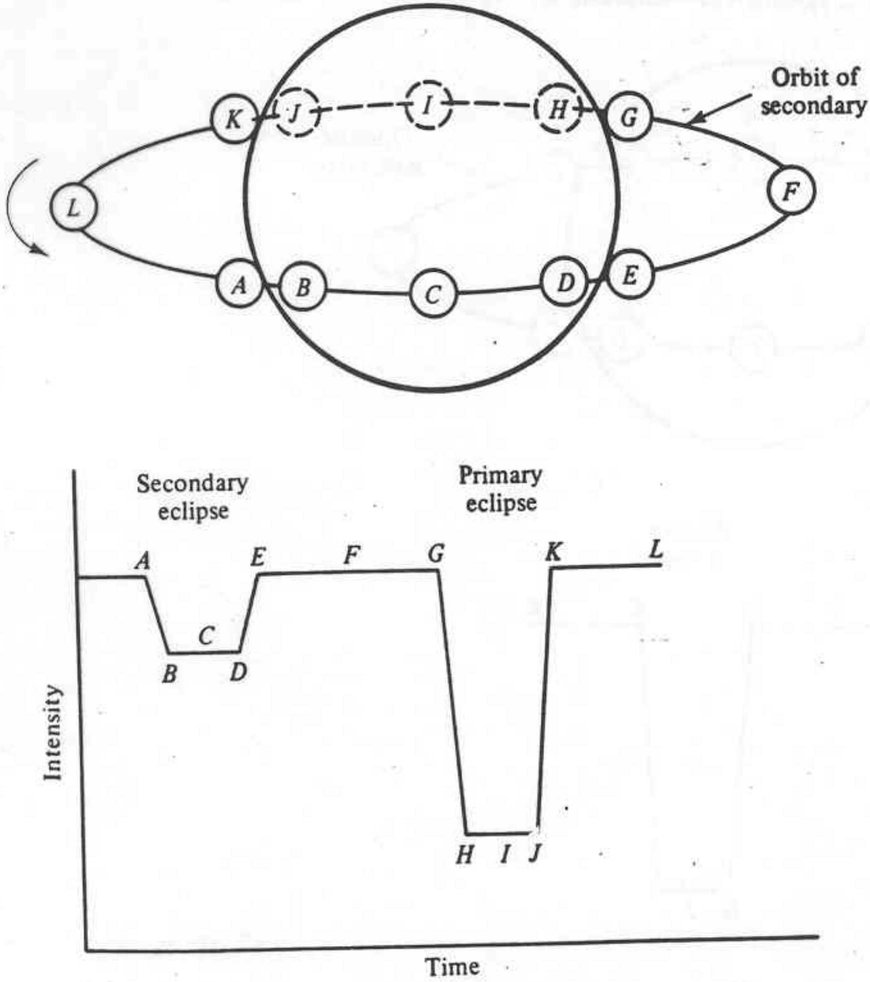
recall $L = \sigma T^4 \, 4\pi R^2$

If we have distance & magnitude m, can get L.

To get T calibrated, need R

- eclipsing binaries
- lunar occultations
- speckle interferometry & AO

Figure 5.15 Stellar sizes determined from eclipsing binaries. The orbit is shown above and the light curve is shown below. The lengths of the eclipses, and the steepness of the sections at the beginning and end of each eclipse (such as A–B and E–D) depend on the sizes of the stars.



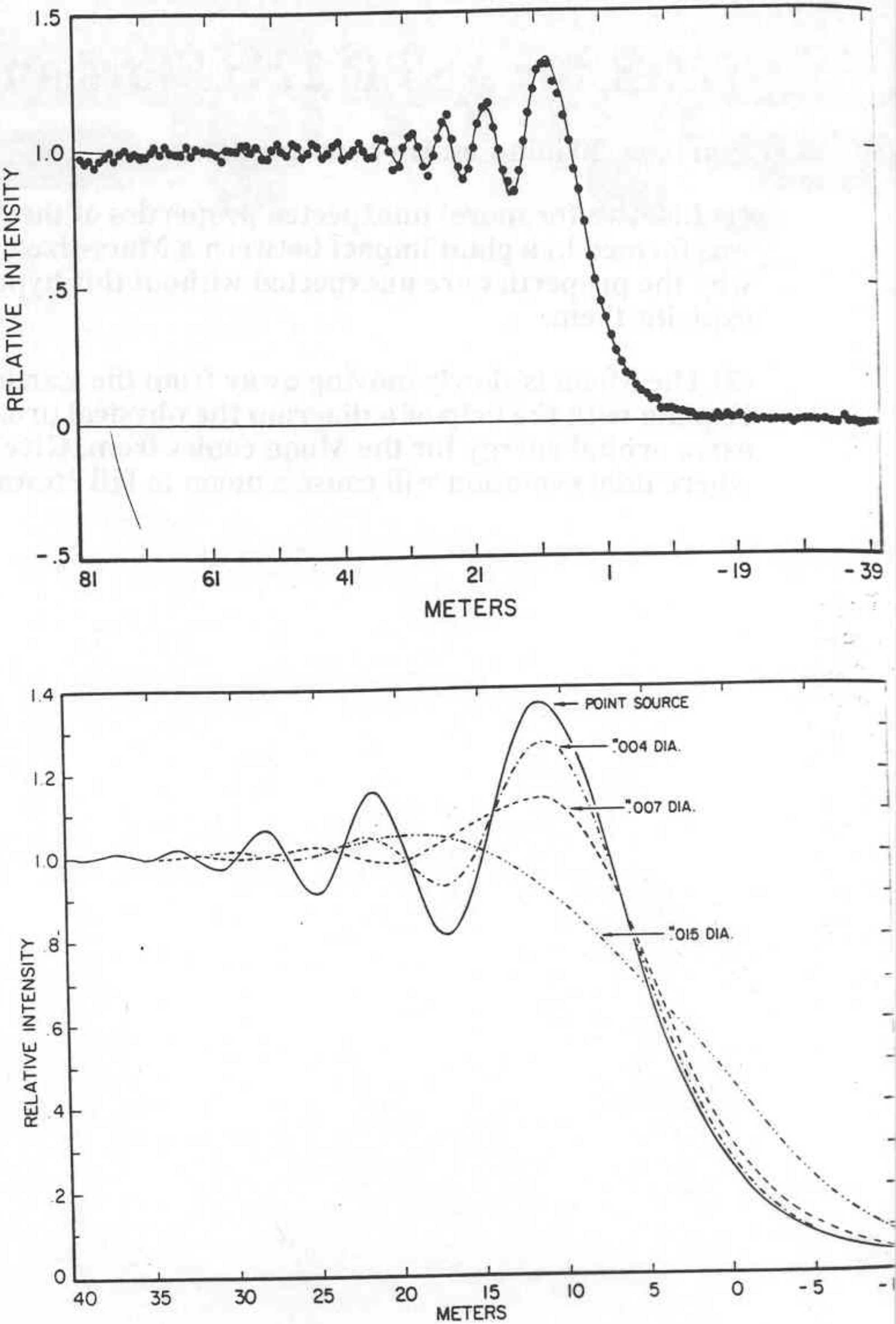


Figure 5.16 (a) Tracing of a lunar occultation of the star β Sco A. On the left we see the star before the occultation, and on the right it is occulted. The wiggles in the curve are due to diffraction effects as the star passes behind the lunar limb. The dots are the actual data, and the smooth curve is the best fit of a theoretical model to the data. (b) Theoretical calculations of what the curves in (a) would look like for stars of different angular sizes.