Versions of the ideal gas law

Temperature

$$PV = NkT$$

pressure volume Boltzmann's const

no of particles

OR $$P = \frac{\rho k T}{\bar{m}}$$

average mass of gas particle

define μ, mean molecular weight

$$\mu = \frac{\bar{m}}{m_H}$$

k/m_H

$$P = \frac{\rho k T}{\mu m_H} = \frac{R}{\mu} \rho T$$

Earth's atmosphere

We can treat the atmosphere as an ideal gas:

$$P = \frac{\rho}{m} kT$$

m = average mass per molecule

Earth's atmosphere is mostly N_2 and O_2

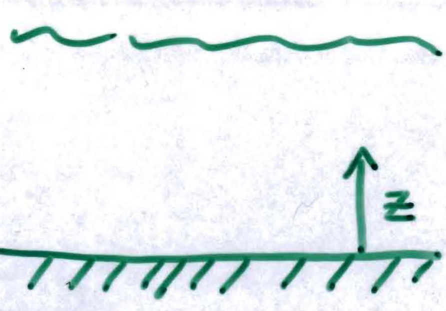
So $m \simeq 29 m_p$

Pressure at surface $= 1$ atm
$= 10^6$ dyn/cm^2

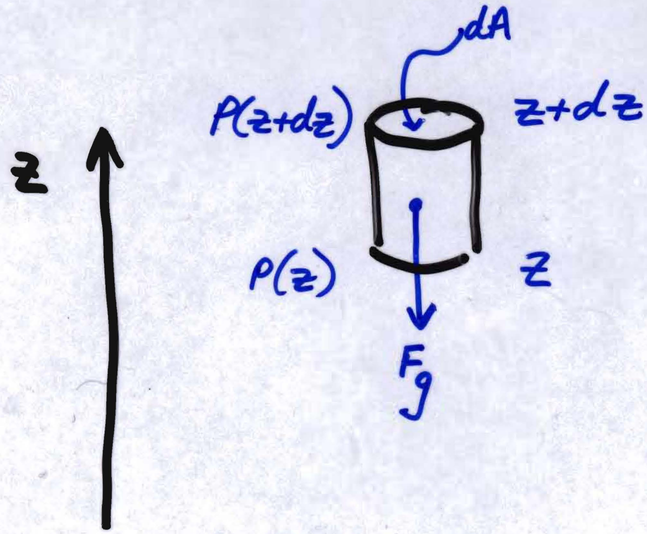
at 300 K, $\rho = 1.1 \times 10^{-3}$ g/cm^3

Pressure gradient is governed by equation of hydrostatic equilibrium

Can treat atmosphere as a plane-parallel layer:



EQN OF HYDROSTATIC $\equiv^m$.



Ideal gas law $P = \frac{R}{\mu} \rho T$

Downward force F_g = mass × accel^ from gravity

$= dm \times g_z$

$= \rho \, dz \, dA \, g_z$

Difference in pressure is $P(z+dz) - P(z)$

Force due to this is $[P(z+dz) - P(z)] \, dA$

Balance : $-\rho g_z \cdot dz \, dA = \left(P(z+dz) - P(z)\right) \cdot dA$

So $$\frac{P(z+dz)-P(z)}{dz} = -\rho g_z$$

or $$\boxed{\frac{dP}{dz} = -\rho g_z}$$ Eqn. of hydrostatic equilibrium

Substitute for ρ from ideal gas law

$$\frac{dP}{dz} = -g \cdot \frac{\mu P}{RT}$$

$$= \frac{-g\mu}{RT} \cdot P$$

If T constant, g const, μ const

call $H = \frac{RT}{g\mu}$ the scale height (dimensions of distance)

$$\frac{dP}{dz} = -\frac{1}{H} P$$

$$\frac{dP}{P} = -\frac{1}{H} dz$$

$$\int_{P_0}^{P} \frac{dP'}{P'} = -\frac{1}{H}\int_{0}^{z} dz'$$

$z=0$ @ "base of atmosphere"

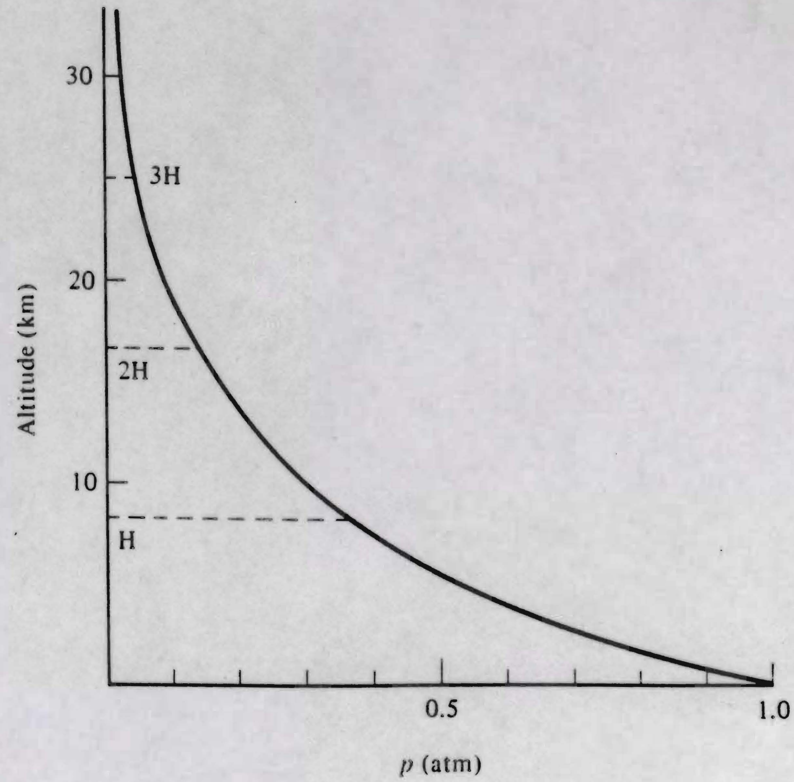
$$\ln\left(\frac{P}{P_0}\right) = -\frac{1}{H} z$$

$$\boxed{P = P_0 \, e^{-z/H}}$$

scale height — distance over which pressure drops by a factor of e (~37%)

— for most planets, of order 10 km

Figure 23.5 Pressure vs. altitude in the Earth's atmosphere. In plotting quantities about the atmosphere, we usually plot altitude on the vertical axis, even though it is the independent variable. The scale height is indicated by H.



For Earth's atmosphere $\bar{m} = 29\,m_p$

$H = 8.7$ km

For hydrogen, $H = 125$ km.

- It turns out that most planets have similar scale heights:

	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
T_{surf}(K)	733	288	215	165*	135*	76*	72*
Albedo	0.75	0.29	0.16	0.34	0.34	0.29	0.31
H (km)	16	8.5	18	18	35	20	19

* Temperature measured at 1bar pressure

from Francis Nimmo, UCSC