VIRIAL THEOREM

Relates potential and kinetic energy of system in equilibrium

$$2\ k.e + p.e = 0$$

$\boxed{Q}$ How might we summarize ~~potent~~ kinetic energy for a giant elliptical? (a quantity we can measure fairly easily)

→ use velocity dispersion σ^2, specially if there is no rotational support.

$\boxed{Q}$ What could we measure that would quantify potential energy?

cf $-\frac{GM}{r}$

Structure of ellipticals – Trends

Elliptical luminosity decreases with radius by the $R^{1/4}$ law:

$$\log I \propto \left(\frac{r}{r_e}\right)^{1/4}$$

Here, effective radius gives a measure of the size of the galaxy (it's the radius containing half the total light)

We can also measure the galaxy's surface brightness: I_e is the mean surface brightness of the galaxy inside r_e (μ_e in magnitudes)

How do size and mean surface brightness correlate with galaxy luminosity?

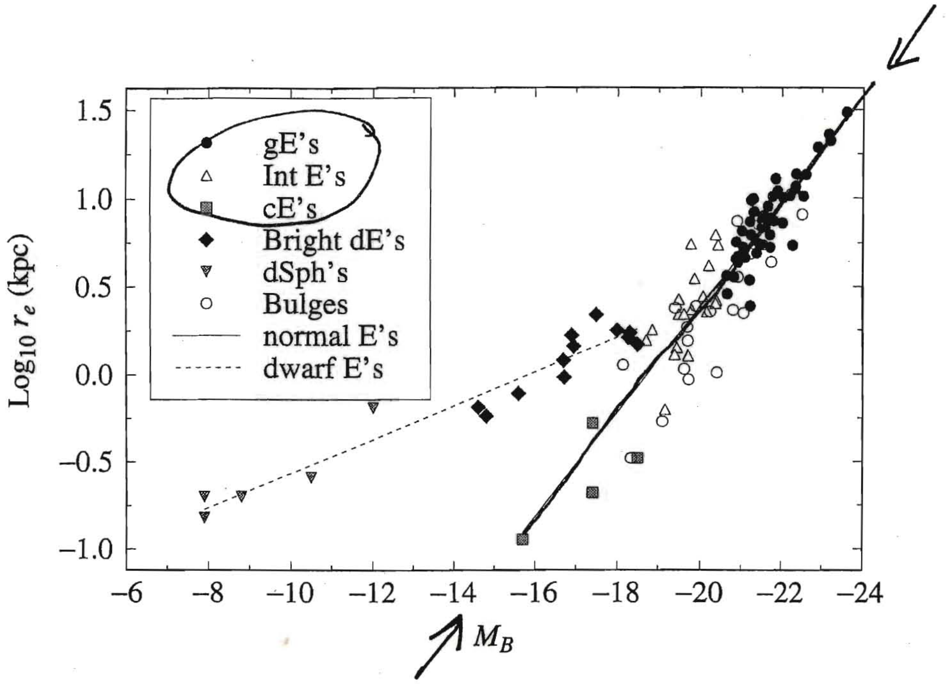

Figure 23.29 Normal ellipticals and dwarf systems define separate sequences of effective radius as a function of M_B. The bulges of spiral galaxies lie along the sequence of normal ellipticals. (Data from Bender et al., *Ap. J.*, *399*, 462, 1992.)

Brighter ellipticals are BIGGER

(note different sequence for dE & dSph galaxies)

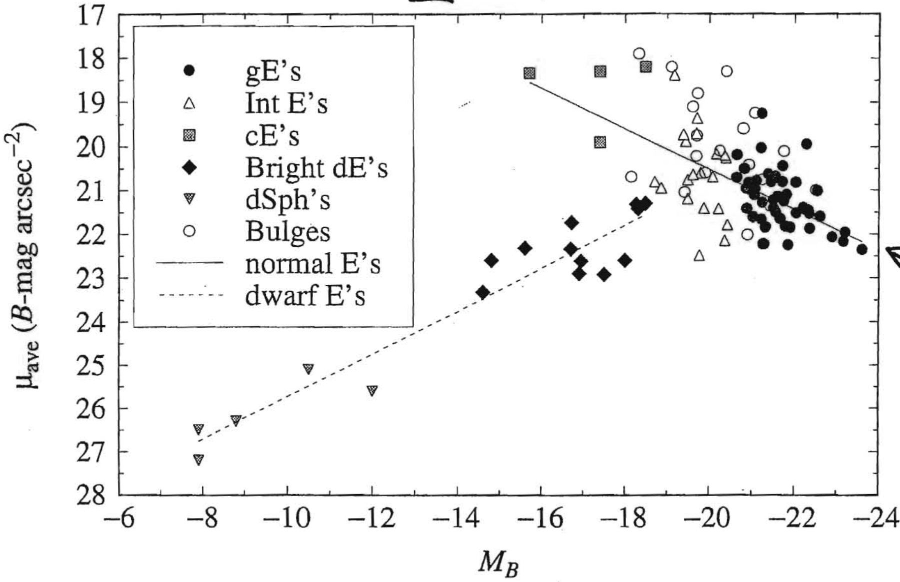

Figure 23.30 Normal ellipticals and dwarf systems define separate sequences of average surface brightness versus M_B. The bulges of spiral galaxies lie along the sequence of normal ellipticals. (Data from Bender et al., *Ap. J.*, *399*, 462, 1992.)

Brighter ellipticals have LOWER (fainter) average surface brightness

More trends for ellipticals

Define L_e = luminosity interior to R_e.

σ_0 = central velocity dispersion

<u>Faber-Jackson</u> relation connects luminosity & kinematics (cf Tully-Fisher relation for spirals)

$$L_e \propto \sigma_0^{\,4}$$

We can define different relations between R_e, surface brightness within R_e ($\langle I_e \rangle$), L_e and σ_0.

The <u>fundamental plane</u> for (giant) elliptical galaxies relates these quantities.

ie $$\log R_e = 0.36 \langle \mu_e \rangle + 1.4 \log \sigma_0$$

measured in B band surf. brightness

$$\langle \mu_e \rangle = -2.5 \log \langle I_e \rangle + \text{const}$$

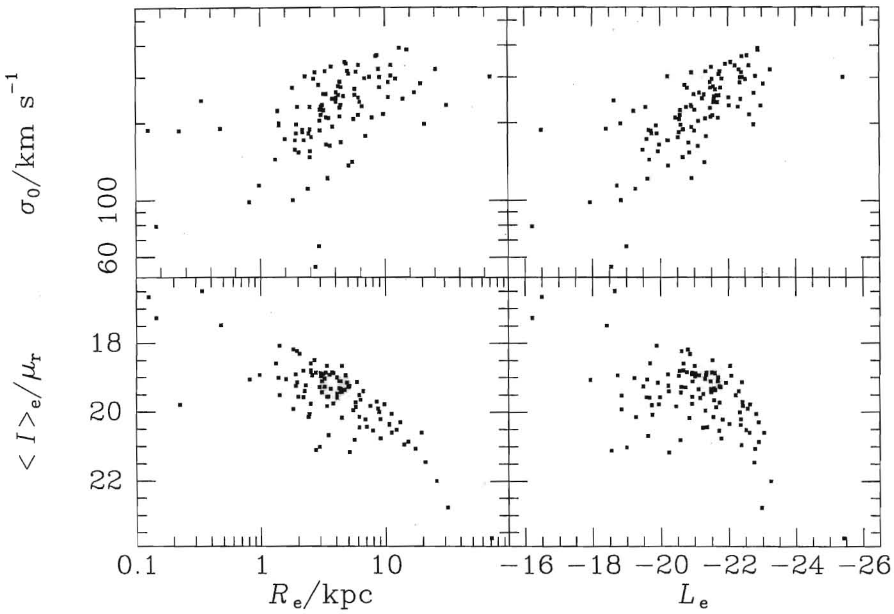

Figure 4.43 Correlations between four shape-independent parameters of elliptical galaxies. The parameters are the effective radius R_e, the mean surface brightness within R_e, $\langle I \rangle_e$, the central velocity dispersion σ_0 and L_e, the luminosity in Djorgovski's G band interior to R_e. The luminosity and the surface brightness are expressed in magnitudes and in magnitudes per square arcsecond, respectively. [From data published in Djorgovski & Davis (1987)]

The lower right panel of Figure 4.43 shows the correlation of which this is the mean regression.

More luminous ellipticals have larger central velocity dispersions. The upper right panel in Figure 4.43 illustrates this correlation, which is called the **Faber–Jackson relation** after its discoverers (Faber & Jackson 1976). Quantitatively one has

$$L_e \sim \sigma_0^4. \tag{4.38}$$

Since σ_0 is correlated with L_e and L_e is correlated with R_e it follows that σ_0 must be correlated with R_e. The top left panel in Figure 4.43 displays this correlation.

Since σ_0 is strongly correlated with line-strengths and colors, the existence of a host of additional correlations involving Mg_2, $B-V$ etc is implied by the top two panels of Figure 4.43. One of the earliest of these to be discovered was the **color–magnitude relation**: Faber (1973) showed that more luminous elliptical galaxies have stronger absorption lines, and Visvanathan & Sandage (1977) showed that more luminous galaxies are redder.

In all the correlations of Figure 4.43 there is **cosmic scatter**; the scatter of the points about the mean relations is larger than can be accounted

Box 4.2: Pr

Given M points $\mathbf{x}^{(\alpha)}$ in
d-dimensional unit vecto
nearly as possible on the
respect to p and the com

subject to the constrair
multipliers this problem

$$0 = \sum_{\alpha=1}^{M} \Big[(\mathbf{n}$$

$$0 = \sum_{\alpha=1}^{M} (\mathbf{n} \cdot$$

where λ is the undeterr
using the last equation t
Then we find that

$$0 = \sum_{\alpha} \Big(\mathbf{n}$$

This equation can be rev

$$A_{ij} \equiv$$

Thus the required vector
A. One may show (see
values taken by S for **n**
the desired **n** is the eige
In order to minimiz
have been scaled such th

for by measurement errors
als between the positions
correlated. For example,
relation in the top right pa
magnitude relation in the
of both the basic correlat
is useful to imagine each

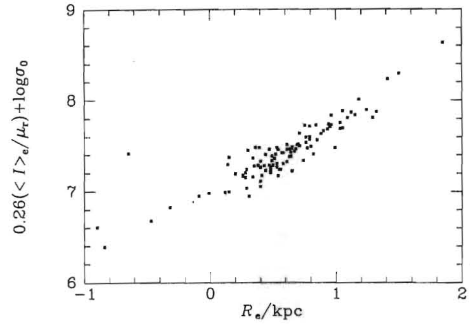

Figure 4.44 An edge-on view of the fundamental plane as defined by the data of Figure 4.43. Note how much narrower is the distribution of points than in either of the left-hand panels of Figure 4.43.

dimensional space. The Cartesian coordinates x_i of a point in this space are the three numbers of the set $(\log R_\mathrm{e}, \langle I\rangle_\mathrm{e}, \log\sigma_0)$, where $\langle I\rangle_\mathrm{e}$ is in units of μ_B.[14] If there were no correlations between the variables, the points of individual galaxies would be fairly uniformly distributed within a cuboidal region of our three-dimensional space. Correlations between the variables will confine the data points to a sub-volume of this cuboid. For example, if two of the variables are genuinely independent and the third dependent on these two, the points will be confined to a plane, while if there is only one independent variable, the points will lie on a line. Our first step in analyzing such data is to ask whether the points are nearly confined to a plane; if we can find a suitable plane, we can further enquire whether the points lie on a line within that plane. In statistics books this type of investigation is called **principal component analysis**. Box 4.2 explains what has to be done.

For the data in Figure 4.43 one finds that the points are as nearly confined to a plane as observational errors allow; in the absence of observational errors, they might lie *precisely* on a plane! If the position vector of a galaxy in our three-space is $\mathbf{g} = (\log R_\mathrm{e}, \langle I\rangle_\mathrm{e}, \log\sigma_0)$, where R_e is measured in kpc, $\langle I\rangle_\mathrm{e}$ in units of μ_B and σ_0 in $\mathrm{km\,s^{-1}}$, then the equation of this **fundamental plane** is $\mathbf{n}\cdot\mathbf{g} = 1$ where $\mathbf{n} = (-0.65, 0.22, 0.86)$.

Naturally one wants to 'see' the fundamental plane. One way to do this is to choose an axis, for example the R_e-axis, as the horizontal axis, and rotate the space about this axis until the plane appears edge-on. In this orientation, the plane's normal, $\mathbf{n}$, lies in the plane of the projection, which is spanned by the unit vector, $\mathbf{e}_R$, that runs parallel to R_e-axis and some other, orthogonal, unit vector $\mathbf{e}$. Thus $\mathbf{n} = \alpha\mathbf{e}_R + \beta\mathbf{e}$, where α and β are suitable numbers. Comparing this with the value of $\mathbf{n}$ given above we see that $\beta\mathbf{e} = 0.22\mathbf{e}_I + 0.86\mathbf{e}_\sigma$. Thus the fundamental plane will be seen edge-on if we plot $\log R_\mathrm{e}$ against $0.22(\langle I\rangle_\mathrm{e}/\mu_B) + 0.86\log\sigma_0$. Actually, it is conventional to plot $0.26(\langle I\rangle_\mathrm{e}/\mu_B) + \log\sigma_0$, which is simply $1/0.86$ times the linear combination we have derived. Figure 4.44 shows this plot. The

[14] L_e need not be included in the set since it is related to R_e and $\langle I\rangle_\mathrm{e}$ by $L_\mathrm{e} = \pi\langle I\rangle_\mathrm{e}R_\mathrm{e}^2$, where L_e is measured in W, I_e in $\mathrm{W\,m^{-2}sterad^{-1}}$ and R_e in m.

Fundamental Plane

Recall the virial theorem: for a system in equilibrium

$$2.\,ke + pe = 0$$

For an elliptical galaxy, particularly one with no rotational support

$k.e. \propto \sigma^2$ stellar velocity dispersion

$p.e. \propto -\frac{GM}{r_e}$

So $\sigma^2 \propto \frac{GM}{r_e}$

$$A.\,\sigma^2 = B.\,\frac{GM}{r_e}$$

Now $M = M/L \cdot L$

surface brightness $I_e = \frac{L_e}{\pi r_e^2}$

$$\text{So} \quad A \cdot \sigma^2 = B \cdot G \left(M/L \cdot \pi r_e^2 I_e \right) / r_e$$

$$\log \sigma^2 = \log(M/L) + \log r_e + \log I_e + \text{const}$$

If M/L is constant (old stellar population) then we would expect a relationship between $\log r_e$, $\log \sigma$ and $\log I_e$ ($= \mu_e$, surface brightness)

<u>This is the fundamental plane.</u>

Q If two ellipticals are merging, would we expect such a relationship to hold?

Q If we observe a population of elliptical galaxies @ $z = 1$ (say in a massive cluster) would the fundamental plane equation still hold? Why or why not?

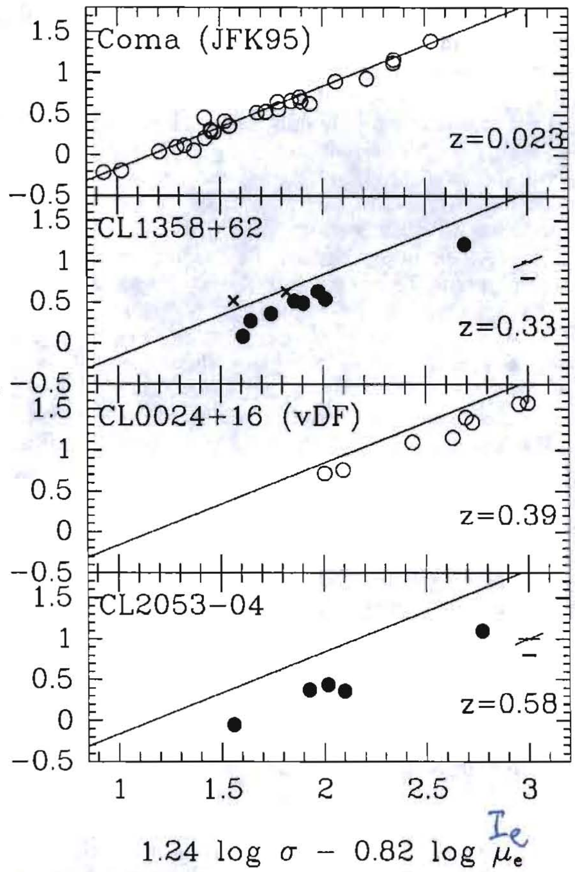

FIG. 2.—Fundamental plane for the four clusters, with the mean fundamental plane for Coma (JFK96) plotted on each panel. Note that the two E+A's in Cl 1358+62 (*crosses*) lie to the left of the mean relation of the "old" Cl 1358+62 early-type galaxies. The previous results of JFK96 and vDF are shown as open circles. The new data are shown as filled circles. Typical random errors (*thin bars*) and systematic errors (*thick bars*) are shown.

$r^{1/4}$-law model could cause systematic errors on the level of $\pm 1\%$ in the combined parameter $r_e \mu_e^{0.82}$.

Another source of uncertainty is due to departures from homology (see, e.g., Capelato, de Carvalho, & Carlberg 1995; Ciotti, Lanzoni, & Renzini 1996). Nonhomology can affect our measurement of evolution through the aperture correction for the velocity dispersions. Jørgensen et al. (1995b) determined the aperture corrections empirically, by using long-slit data on nearby galaxies. They found no strong effect out to an effective radius. Therefore these aperture corrections are likely to be appropriate for most of our galaxies. However, for the smallest galaxies, this correction is more uncertain and may require future observations of velocity dispersion profiles to large radii in a broad sample of nearby galaxies (see, e.g., Corollo et al. 1995).

3. THE FUNDAMENTAL PLANE IN Cl 1358+62 AND MS 2053−04

The fundamental planes for the clusters are shown in Figure 2 along with those for Cl 0024+16 (vDF) and Coma (JFK96). We use the coefficients for the fundamental plane determined by JFK96 from a large sample of 225 early-type galaxies in 10 nearby clusters. The figure shows clearly that a well-defined fundamental plane exists, *despite the fact that the galaxies in the intermediate-redshift clusters were chosen without morphological information*. Furthermore, the sample is large enough to derive the scatter about the Coma fundamental plane. We find surprisingly low rms scatters in log r_e of ± 0.064, ± 0.065, ± 0.060, and ± 0.072 for Coma, Cl 1358+62, Cl 0024+16, and MS 2053−04, respectively. The galaxies also show a large offset from the Coma relation, due mainly to cosmological surface brightness dimming.

One interesting question is whether the coefficients of the fundamental plane are the same in higher redshift clusters, i.e., are the luminous and less luminous galaxies evolving at the same rate? However, the current sample is too small to provide a definitive answer. The weak indication that the slope is flatter when the distant galaxies are taken together needs to be verified with larger samples before any conclusions should be made (see also vDF).

We determined the mean M/L_V ratio for each cluster directly from the fundamental plane zero point, adopting the slopes of the fundamental plane of JFK96 and $q_0 = 0.05$. The resulting evolution of M/L_V is shown in Figure 3. The errors are taken from § 2.3 and have been added in quadrature. Weighting the individual galaxies by their random errors does not change the results significantly.

Clearly, the M/L_V ratio is lower at higher redshift, consistent with evolution of the stellar populations. We have drawn simple, single-burst model predictions in the same plot, adopting formation redshifts $z_{\rm form}$ of infinity and $z_{\rm form} = 1$. The current data are not consistent with the predictions for coeval populations that have formed recently. More data are needed to test whether more complex models with recent star formation can be accommodated (see, e.g., Franx & van Dokkum 1996; Poggianti & Barbaro 1996).

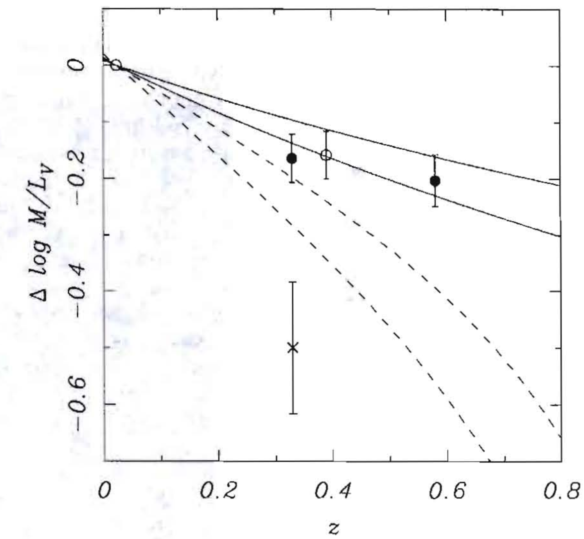

FIG. 3.—Mean M/L_V offsets with redshift, for $q_0 = 0.05$. The area enclosed by solid lines corresponds to single-burst models with $z_{\rm form} = \infty$ and a range of initial mass functions. The region marked by dashed lines corresponds to the equivalent $z_{\rm form} = 1$ models. The previous results of JFK96 and vDF are shown as open circles. The new data are shown as filled circles. The cross marks the M/L_V offset derived from the two E+A galaxies in Cl 1358+62. The errors were estimated by adding the random and systematic errors in quadrature.

4. DISCUSSION

We have measured structural parameters and central velocity dispersions for galaxies in two clusters at intermediate redshift, Cl 1358+62 at $z = 0.33$ and MS 2053−04 at $z = 0.58$.

Making ellipticals with mergers

Only quite recently have simulations become realistic enough to include gas (& feedback) in merger sims.

→ Comparison of mergers with stars only & with significant amounts of gas show different amounts of rotational support in remnants

→ 'Tilt' of fundamental plane (deviation from predictions of virial theorem) can be reproduced via gas-rich mergers.

→ ~~Rotation~~ Elliptical galaxies made via gas-rich AND gas-free mergers (dissipationless) Most massive Es made via dry (dissipationless) mergers. Less massive ones need gas in simulations to reproduce properties.

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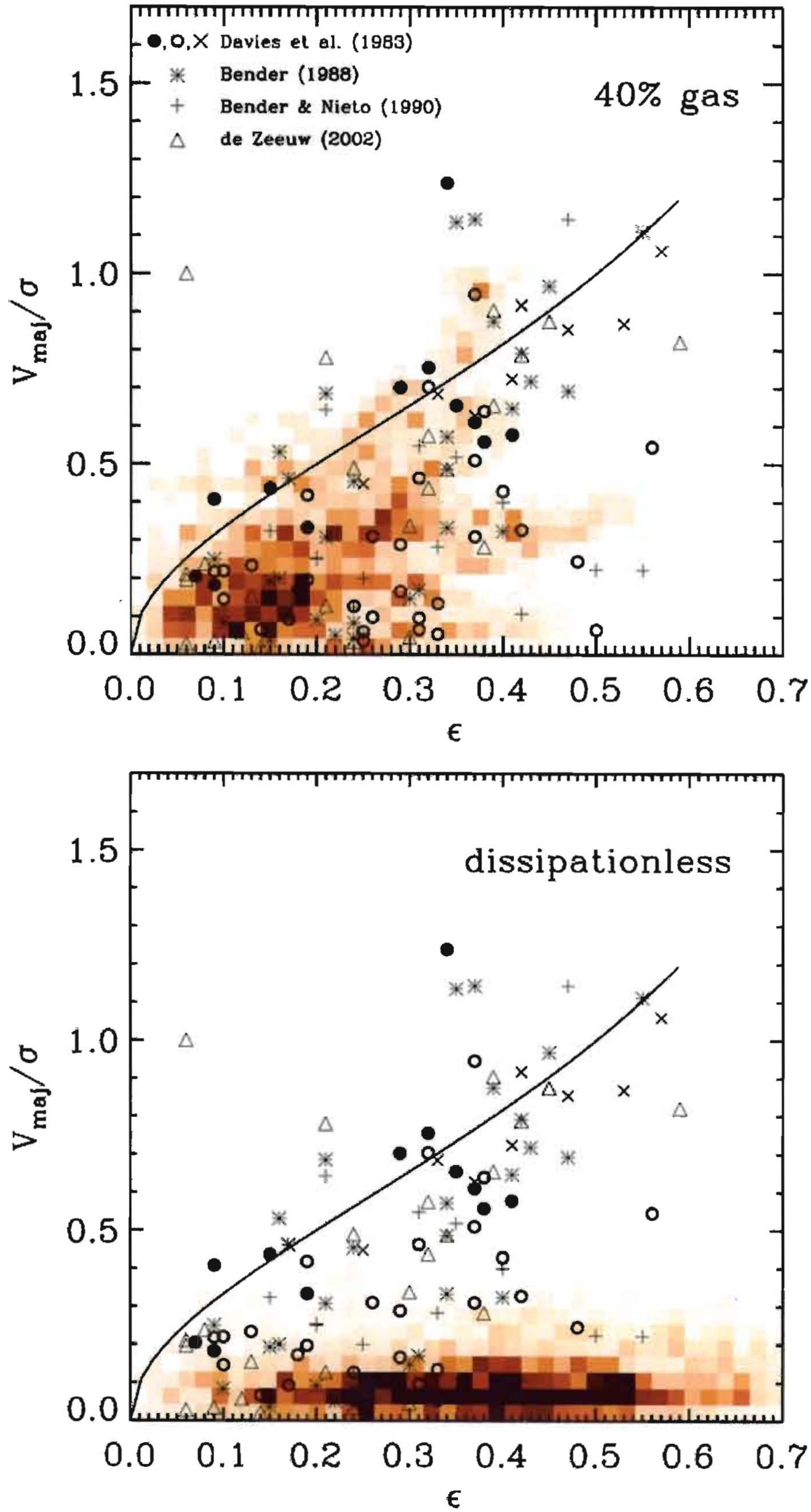
V_{maj}/σ vs. ellipticity diagram for dissipational (40% gas) and dissipationless merger remnants. V_{maj} is the maximum rotation speed measured in a slit along the major axis, σ is the velocity dispersion averaged within half of an half-mass radius, and the ellipticity is measured at the half-mass isophote. Further details can be found in § 2.2. The solid line in both plots is that expected for an oblate isotropic rotator (Binney 1978). Overplotted are data from observed ellipticals from Davies et al. (1983), Bender (1988), Bender & Nieto (1990), and de Zeeuw et al. (2002).



Cox et al (2006)