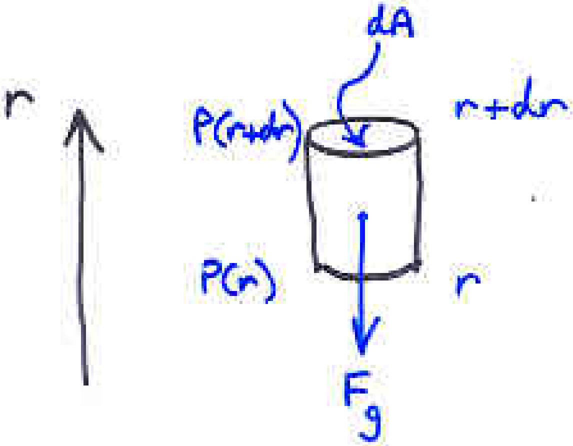
HYDROSTATIC EQUILIBRIUM

The Sun is balanced between :

- collapsing under its own gravity, &
- expanding due to outward pressure of hot gas.



Consider a volume element of gas.

Ideal gas law $\quad P = \frac{R}{\mu} \rho T$

P pressure, T temperature

ρ density, μ molecular weight, R gas constant

- Downward force F_g = mass $\times$ accel^n from gravity

$$= dm \times g_r$$

$$= \rho \, dr \, dA \cdot g_r$$

- Difference in pressure is

$$P(r+dr) - P(r)$$

- Force due to this is $[P(r+dr) - P(r)]\, dA$

Balance forces : gravity vs pressure

$$-\rho \, dr \, dA \cdot g_r = (P(r+dr) - P(r))\, dA$$

So $-\rho g_r \, dr = P(r+dr) - P(r)$

$$\frac{P(r+dr) - P(r)}{dr} = -\rho g_r$$

$$\boxed{\frac{dP}{dr} = -\rho g_r}$$ eq^n of hydrostatic $\equiv^m$.

Equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho(r)\, g(r)$$

Use this eqn to estimate the central pressure of the Sun by considering whole Sun as one shell

$\Delta P = P_c$ ($P = 0$ at edge)

$\Delta R = R$

$$\frac{\Delta P}{\Delta R} = -\frac{GM}{R^2} \cdot \rho$$

ρ is average density $\simeq \dfrac{M}{R^3}$

$$\Delta P = P_c = \frac{GM}{R^2} \cdot \rho \cdot R$$

$$\simeq \frac{GM}{R} \cdot \frac{M}{R^3} = \frac{GM^2}{R^4}$$

substituting for G, M & R we get $P_c \simeq 10^{16}$ dyn/cm^2

Equation of state for ideal gas P, ρ, T

$$P = \left(\frac{\rho}{m}\right) kT \qquad m \text{ is mass per particle}$$

Use this & estimate of central pressure to get an estimate of the central temp. of the Sun

H completely ionized $\Rightarrow m = \frac{1}{2} m_p$

$$T_c = \frac{m P_c}{\rho k}$$

$$= \frac{1}{2} m_p \cdot P_c \cdot \frac{\frac{4}{3}\pi R^3}{M_\odot} \cdot \frac{1}{k}$$

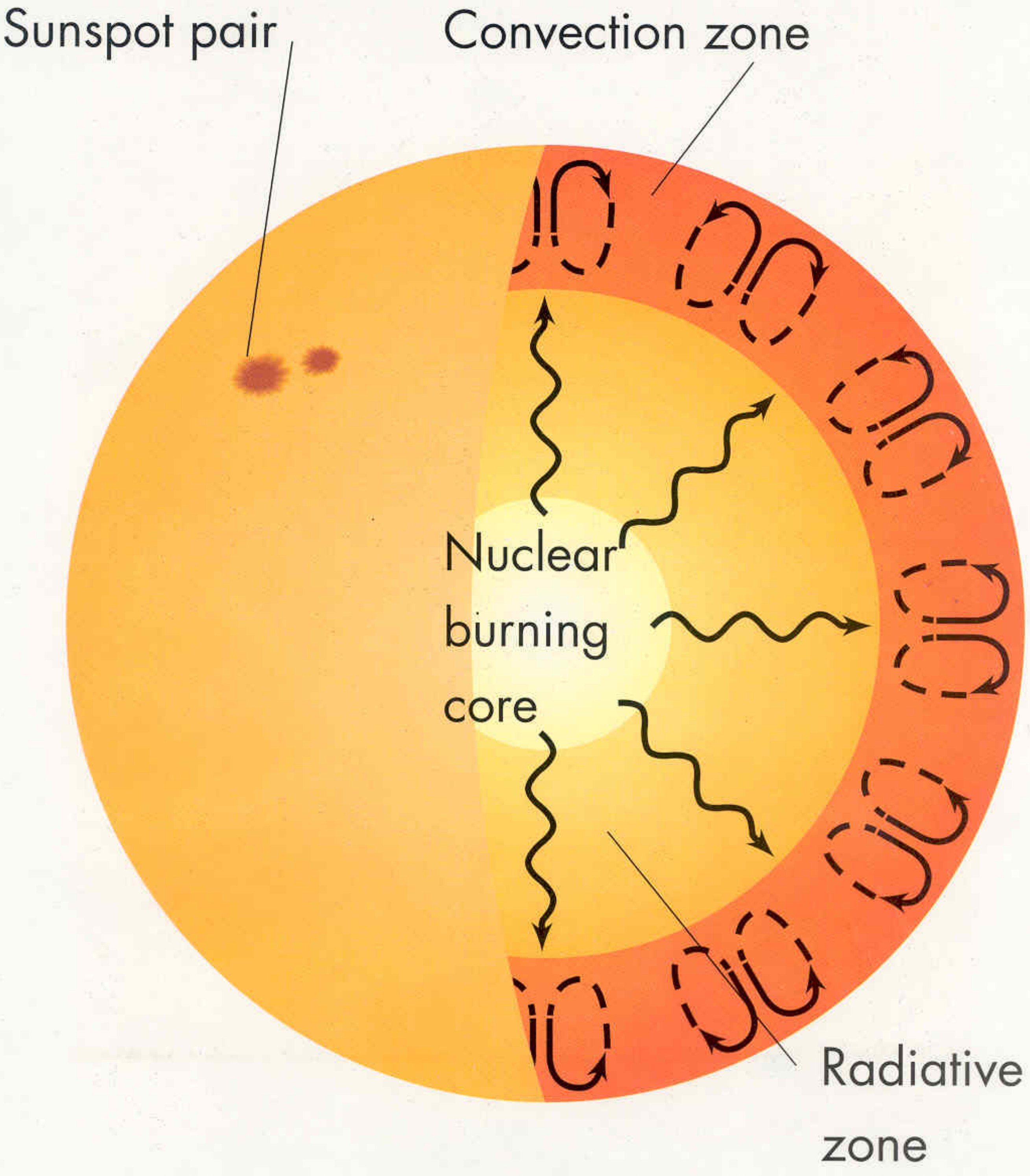
$$= 4.4 \times 10^7 \text{ K}$$

$\boxed{Q.}$

What are some everyday examples of hydrostatic equilibrium?

$\boxed{Q}$ Work out why the Sun doesn't explode, but just keeps burning at a roughly constant rate for billions of years

Energy flow from the Sun's core to its surface (Fig. 10-3)



RADIATIVE TRANSFER

How does the energy get out?

Nuclear reactions give off energy in core as γ-rays.

Thru most of Sun's interior, these are absorbed and re-emitted by the atoms, and are degraded to lower energy in the process.

This is a slow process, called a "random walk" because the emitted photon may move in any direction

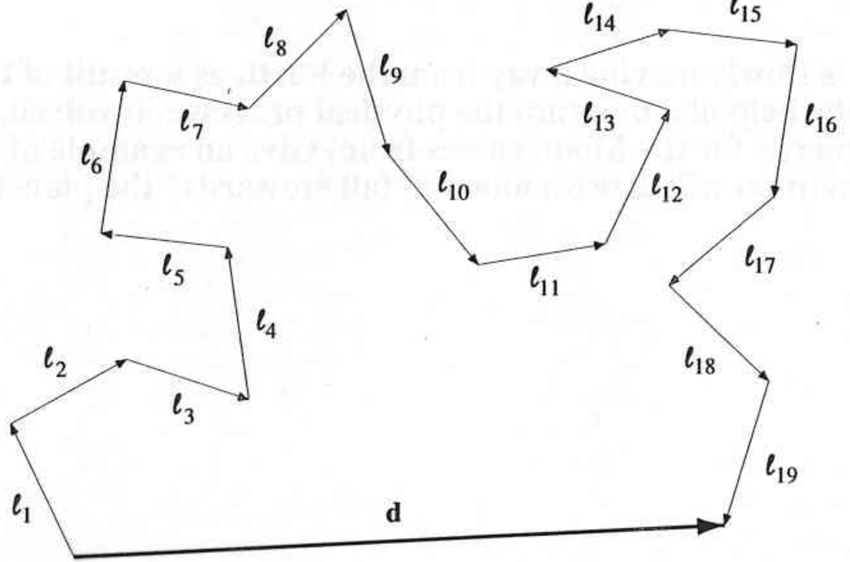
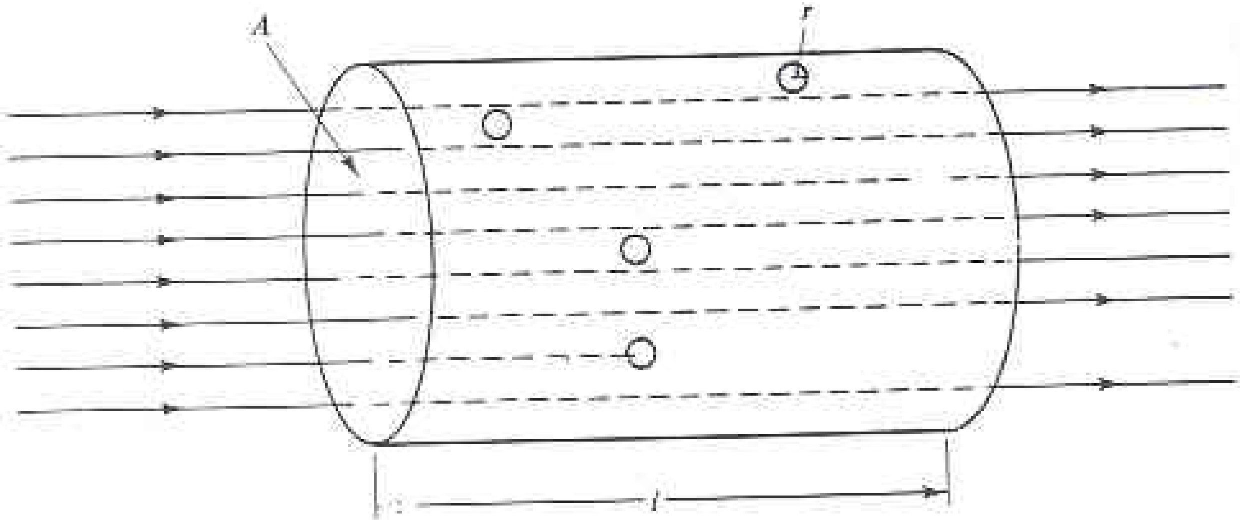


Figure 9.11 Displacement $\mathbf{d}$ of a random-walking photon.

Optical depth

Transfer of radiation through an absorbing medium (atmosphere of star, Earth's atmosphere)

Simple model: absorbers are spheres, radius r



Cross-section for absorption σ

In this case, we put σ = cross-sectional area of sphere

$$\sigma = \pi r^2$$

(σ is used in a more probabilistic sense in quantum mechanics, nuclear physics, etc)

Consider a cylinder: length l

area A

n spheres / unit volume

Volume of cylinder $= lA$

Total no of spheres $N = nlA$

IF spheres dont shadow each other

(ie $\sigma_{tot} << A$)

Total cross-sectional area seen by incoming radiation $\sigma_{tot} = N\sigma$

$= nlA\sigma$

Fraction of light absorbed is fraction covered by spheres $= \dfrac{\sigma_{tot}}{A}$

$= \dfrac{nlA\sigma}{A}$

$= nl\sigma$

Optical depth $\boldsymbol{\tau}$ (dimensionless)

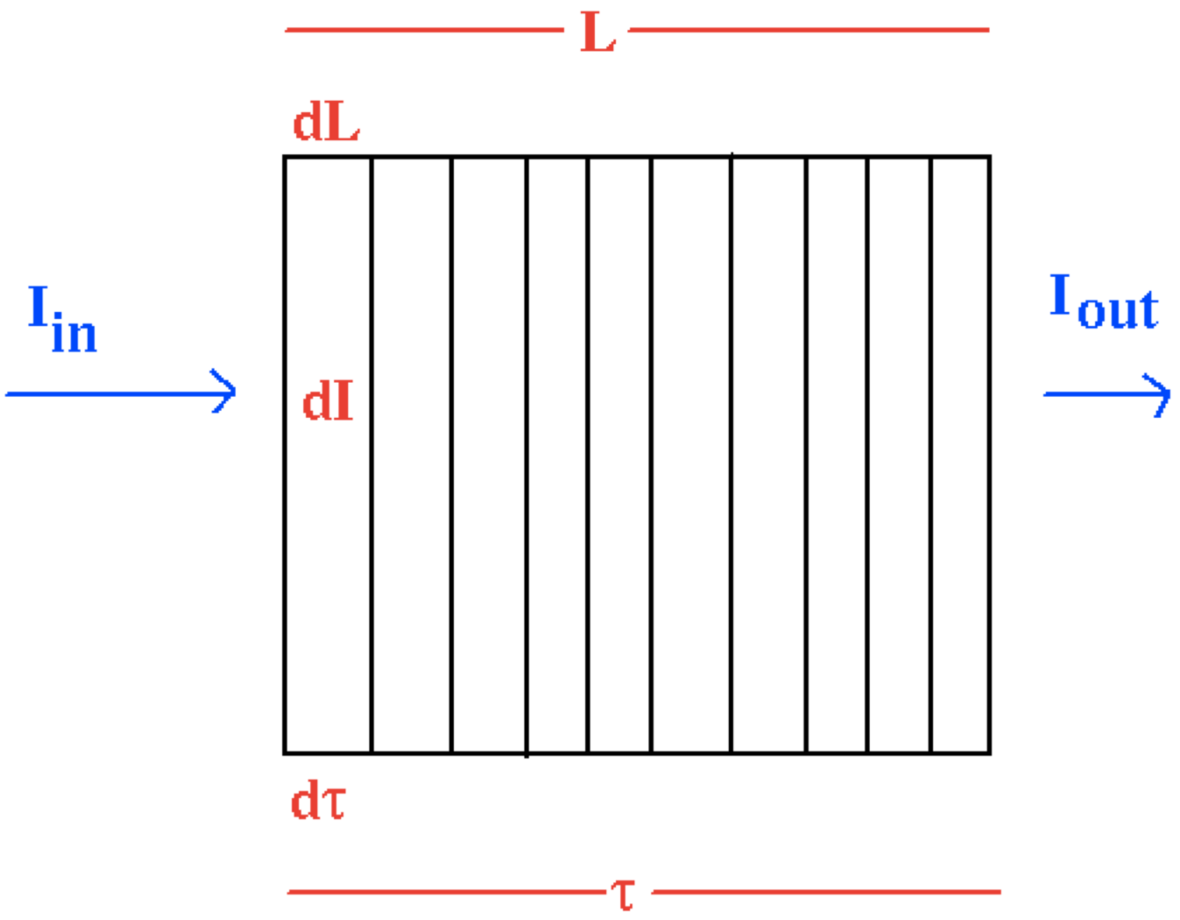
$$\tau = n\, l\, \sigma$$

n is number density (1/cm^3); l is length (cm) and σ is cross section (cm^2)

Optical depth $\boldsymbol{\tau = n\, l\, \sigma}$

Think of this from the point of view of the photon …
not only how far it travels but what it encounters

We can see down to about optical depth = 1 in the Sun, for example



Prof Mihos has this nice diagram in his 221 notes which shows that astronomers will use τ instead of length

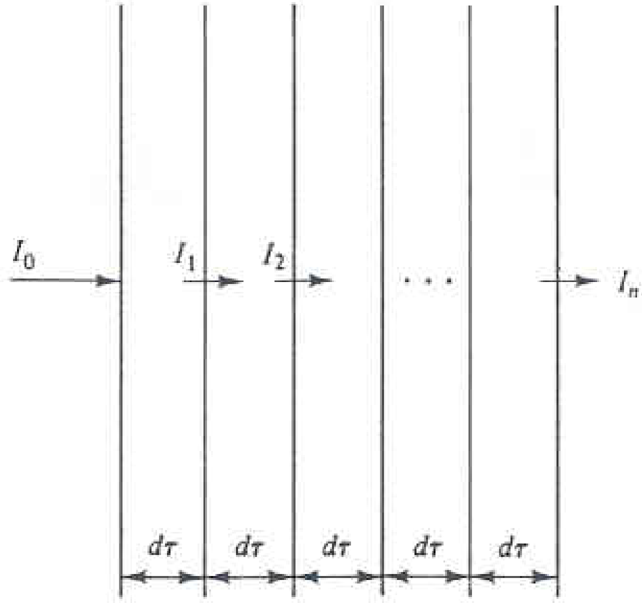
This was derived assuming few absorptions

$(\sigma_{tot} \ll A$

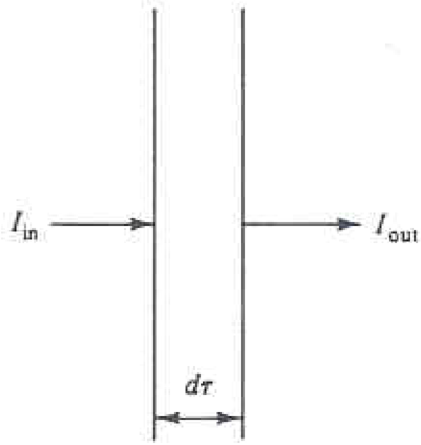
ie $n l A \sigma \ll A$

$n l \sigma \ll 1$

$\tau \ll 1)$



What if you have many?



Divide into small layers with $\tau \ll 1$ for each

$$dI = I_{out} - I_{in}$$

$$= -I\, d\tau$$

τ is fraction absorbed

IF $\tau \ll 1$

$$\frac{dI}{I} = -d\tau$$

negative since intensity ↓

Integrate this : τ' goes from 0 to τ

I' goes from I_0 to I

$$\frac{dI'}{I'} = -d\tau'$$

$$\int_{I_0}^{I} \frac{dI'}{I'} = -\int_0^{\tau} d\tau'$$

$$\left[\ln I'\right]_{I_0}^{I} = -\left[\tau'\right]_0^{\tau}$$

$$\ln I/I_0 = -\tau$$

$$I = I_0 e^{-\tau}$$

So for more absorptions, τ is not fraction absorbed – radiation coming out goes exponentially with τ

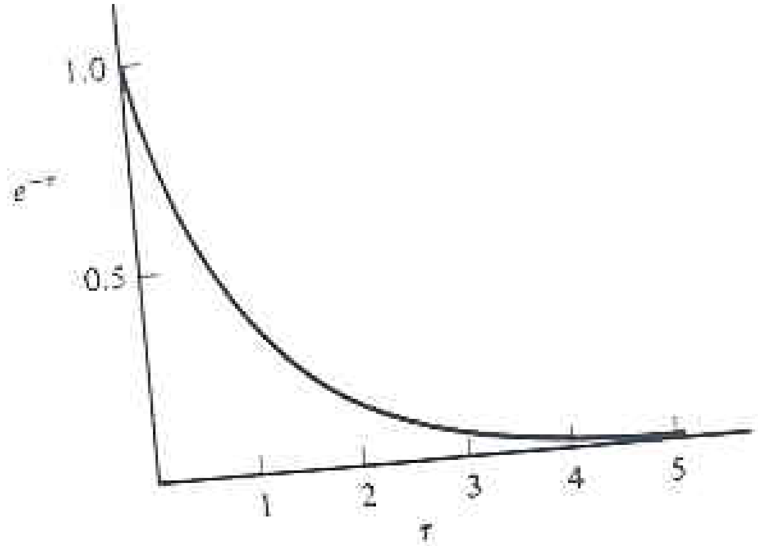


Figure 6.5 $e^{-\tau}$ vs. τ, showing the falloff in transmitted radiation as the optical depth increases. Note that the curve looks almost linear for small τ. For large τ, it asymptotically approaches zero.

For $\tau \ll 1$ $\quad e^{-\tau} \simeq 1-\tau$ *

So $\quad I = I_0 (1-\tau)$

and τ is fraction absorbed

— Maximum absorbed has to be 1 ! —

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

- Absorption coefficient $\chi_\lambda = n\sigma_\lambda$

  number (fraction) of absorptions per unit length, since it is τ_λ / L

- Mean free path = length between absorptions

$$= \frac{1}{\chi_\lambda}$$

$$= \frac{1}{n\sigma_\lambda}$$

In a typical gas @ room temperature & pressure

molecules are of size $\sim 10^{-8}$ cm (1 Å)

mean free path (for collisions) $\sim 10^{-5}$ cm

ie 1000 × diameter of molecule

In the real case of radiative transfer through the Sun, we need to consider emission as well as absorption in each layer.

In general

"Optically thin" $\tau \ll 1$ (most light gets thru)

"Optically thick" $\tau \gg 1$ (most light absorbed)

Examples of optically thick :

Earth's atmosphere in far-UV

Interior of Sun

Convection:

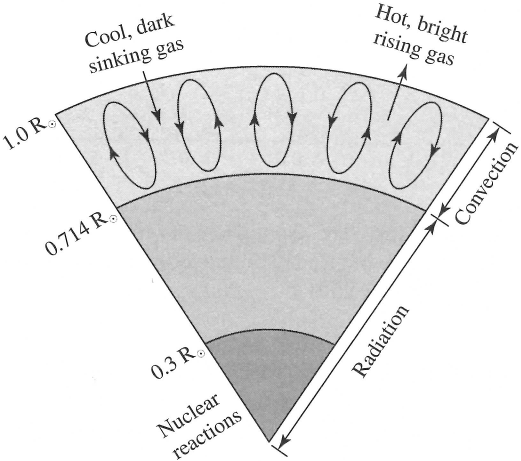
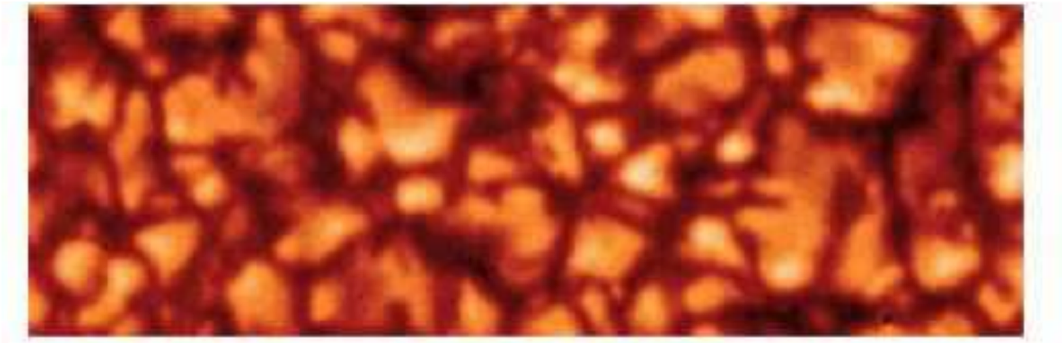


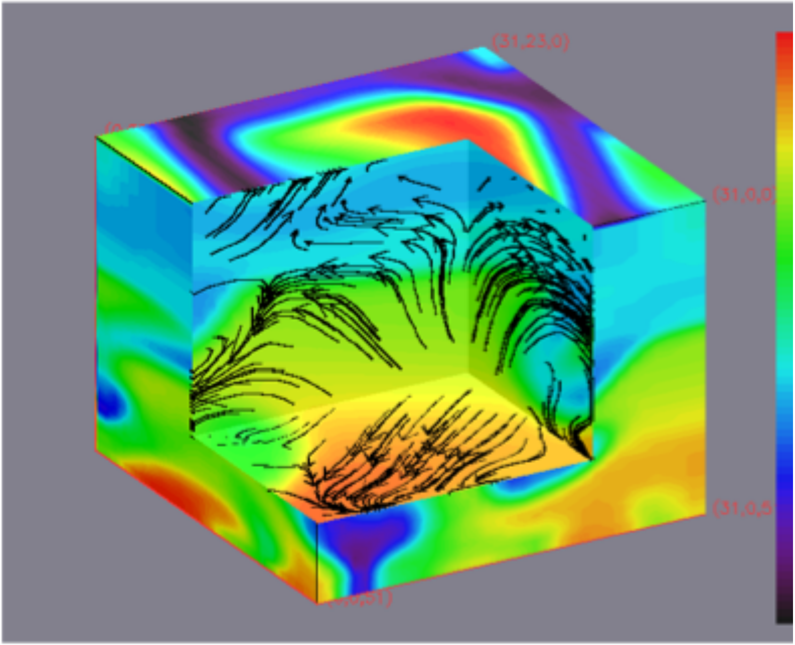
FIGURE 11.2 A schematic diagram of the Sun's interior.

Granulation shows convection:



G-band SST solar image
(Lites et al. 1990)

3D RHD models necessary



Nordlund et al. (2009)

Q

We can measure many of the nuclear reaction rates, estimate temperature & density at surface of Sun, & get a model for the Sun's structure & energy output.

Think of as many observations as we could possibly make of the Sun

Give ≥3 ways we could test the model

- Sun's photospheric properties
- Helioseismology
- Solar neutrinos
- lifetimes of Sun-like stars