ASTR 221 Problem set 6 – Solutions

Q 9.14 It is very difficult to directly observe the nucleosynthesis in the center of a star because it is optically thick: photons produced in the center take many thousands of years to get out. By contrast, the neutrinos get out very easily & so can give constraints on reactions in the core.

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Q 9.15 Gravitational collapse works well as an energy source, but not for billions of years.

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P 9.2 (a) Gravitational potential energy

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

Radius $= 10\,pc = 3.09 \times 10^{19}$ cm

$\rho = 10^3$ H atoms per $cm^3 = 1.67 \times 10^{-24} \times 10^3\ g/cm^3$.

Volume $= \frac{4}{3}\pi R^3$ So since $\rho = M/V$

mass $M = \rho . V = 1.67 \times 10^{-21}\ g\,cm^{-3} . \frac{4}{3}\pi \left(3.09 \times 10^{19}\right)^3 . cm^3$

$= 2.06 \times 10^{38}\ g.$

So $U = -\frac{3}{5} \times \frac{6.67 \times 10^{-8} \text{ dyne cm}^2 g^{-2} \, (2.06 \times 10^{38} g)^2}{3.09 \times 10^{19} \text{ cm}}$

$= -5.1 \times 10^{49}$ erg

(b) kinetic energy $= \frac{3}{2} \frac{M}{m} kT$

$= \frac{3}{2} \frac{2 \times 10^{38} g}{1.67 \times 10^{-24} g} \cdot 1.38 \times 10^{-16} \text{ erg/K} \cdot 10 \text{ K}$

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$= 2.4 \times 10^{47}$ erg

or 2.4×10^{40} J

P10.5 (a) Momentum delivered per sec $= \frac{dP}{dt} = \frac{1}{c} \frac{dE}{dt} = \frac{L}{c}$

luminosity L is $10^2 L_\odot$ so

$\frac{dp}{dt} = \frac{100 \times 4 \times 10^{33} \text{ erg/sec}}{3 \times 10^{10} \text{ cm/s}}$

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$= 1 \times 10^{25}$ g-cm/s^2 or dyne

(b) $F = \frac{G m_1 m_2}{R^2} = \frac{6.67 \times 10^{-8} \text{ dyne cm}^2 g^{-2} \cdot 0.1 \times (2 \times 10^{33} g.)^2}{(100 \times 6.96 \times 10^{10} \text{ cm})^2}$

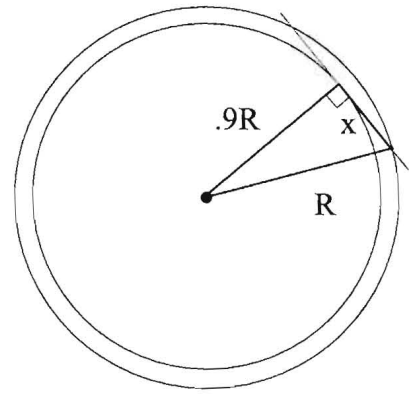
$= 5.5 \times 10^{32}$ dyne

P 10.7 - (a) The shortest path is 2(0.1 R) = 0.2 R

The longest path is 2x, where x is the short side of a right triangle where the other sides are R and 0.9R ; so $x^2 = R^2 - (0.9R)^2$; x = .44 R; 2x = 0.88 R.

Ratio of longest to shortest is 4.4

(b) Longest path corresponds to more emission, so nebulae appear as circular shells.



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P 10.10 Escape speed $= \sqrt{\frac{2GM}{r}}$

Mass $= 1\, M_\odot = 1.99 \times 10^{33}$ g.

So for a white dwarf of one solar mass, use the mass-radius relation from class notes

$R = 5.5 \times 10^8$ cm

So escape speed $= \frac{\sqrt{(2 \times 6.67 \times 10^{-8}\ \text{dyne cm}^2\,\text{g}^{-2} \times 1.99 \times 10^{33}\ \text{g})}}{\sqrt{5.5 \times 10^8\ \text{cm}}}$

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$= 6.9 \times 10^8$ cm/s for white dwarf

and 6.9×10^7 cm/s for Sun.

$\boxed{P10\text{-}11}$ (a) From example 10-2, white dwarf density is of order $10^6\ g/cm^3$.

If we have positive ions with charge Z, equation 10-10 gives that $n_e = \frac{Z}{A}\frac{\rho}{m_p}$ $\longrightarrow$ electron number density

$Z/A \sim 0.5$ (roughly half nucleons are protons)

so $n_e = 0.5 \times 10^6\ g\,cm^{-3} / 1.67 \times 10^{-24}\ g$

$= 3 \times 10^{29}\ cm^{-3}$

$$\Delta x = \frac{1}{(n_e)^{1/3}} = 1.5 \times 10^{-10}\ cm$$

(b) momentum uncertainty : $\Delta x \Delta p \sim h/2\pi$

so $$\Delta p = \frac{6.62 \times 10^{-27}}{2\pi} \cdot \frac{1}{1.5 \times 10^{-10}}$$

$= 7 \times 10^{-18}\ g\text{-}cm/s$

(c) Remembering that the electrons are moving relativistically

$$p = \gamma m v = \frac{mv}{(1 - v^2/c^2)^{1/2}}$$

This gives $v^2 = p^2 / (m^2 + p^2/c^2) = 4.9 \times 10^{-35} / \left((9.1 \times 10^{-28}\ g)^2 + .54 \times 10^{-55}\right)$

So $v = 7.5 \times 10^9$ cm/s (significant fraction of c)

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$\boxed{P\ 11.4}$ (b) Gravitational force is $\frac{GmM}{r^2}$

Tidal force $= -\frac{2GmM}{r^3}$ (ie $\frac{dF}{dr}$)

your mass 1.5 $M_\odot$ neutron *

This is $-2 \cdot \frac{6.67 \times 10^{-8} \cdot 7.5 \times 10^{4} g \, (1.5 \times 2 \times 10^{33} g)}{(1.5 \times 10^{6} \text{ cm})^3}$

$= \sim 10^{13}$ dyne/cm

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Standing, height is ~ 200 cm, so force is

1.78×10^{15} dyne

Prone, say 1/4 of this (50 cm) so 4.4×10^{14} dyne

$\boxed{P\ 11.6}$ Typo in question: put initial radius = r_i, final r_f

Kinetic energy of rigid body $= \frac{1}{2} I \omega^2$

for a rotating sphere $I = 2/5\, Mr^2$

So ke $= Mr^2\omega^2$

If angular momentum conserved $r_i^2 \omega_i = r_f^2 \omega_f$

So $\omega_f = \dfrac{r_i^2}{r_f^2}\,\omega_i$

So $\dfrac{ke_{final}}{ke_{initial}} = \dfrac{\cancel{m}\, r_f^2\, \omega_f^2}{\cancel{m}\, r_i^2\, \omega_i^2}$

$= \dfrac{\omega_f^2}{\omega_i^2} \cdot \dfrac{\omega_i}{\omega_f}$

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$= \dfrac{\omega_f}{\omega_i}$

~~Difference is radiated away~~

Since star is rotating faster, it has gained k.e at the expense of gravitational p.e.