

ASTR 221 Problem Set 1 - Solutions

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1 (80 points)

2 (a) $m = \frac{4}{3} \pi R^3 \rho$

$$\begin{aligned} \frac{dm}{dt} &= \frac{d}{dt} \left(\frac{4}{3} \pi R^3 \rho \right) = \frac{4}{3} \pi \rho \frac{dR^3}{dt} && \text{(assuming density is unchanged)} \\ &= \frac{4}{3} \pi \rho \cdot 3R^2 \frac{dR}{dt} \end{aligned}$$

20 (from notes) $= \rho_s v R^2 \left(1 + \frac{8G\rho R^2}{v^2} \right)$

So $4\pi\rho \frac{dR}{dt} = \rho_s v \left(1 + \frac{8G\rho R^2}{v^2} \right)$

$$\frac{dR}{dt} = \frac{\rho_s v}{4\pi\rho} \left(1 + \frac{8G\rho R^2}{v^2} \right)$$

(b) If left-hand term dominates (large relative velocity, low density) then radius increases linearly with time, and growth rate does not depend on object's size. Called 'orderly growth'.

20 If right-hand term dominates (lower velocity, larger ρ) then large objects will grow faster than small ones, called 'runaway growth'.

Density of planetesimal swarm ρ_s affects both cases equally, simply scaling the timescale for growth.

volatile

(3) From notes, most common elements (excluding noble gases) are H, C, N and O.

This gives compounds CH_4 , NH_3 , H_2O

(CO_2 is less common because it has two 'metals')

From the equilibrium condensation diagram in notes, we see that at a pressure of 10^{-2} bar these three molecules have condensation temperatures $\sim 100 - 150$ K.

This corresponds to a radius range of $\sim 1 - 2$ AU.

Given that Mars is at 1.5 AU and Jupiter is at 5 AU from the Sun, the numbers from the models are a bit low.

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(2)