

White dwarfs

Final stage of evolution of low-mass stars (central star of planetary nebula becomes white dwarf)

~ size of Earth

Q Why do you think the error bars are so large in the top part of the diagram?

Types of white dwarfs

temp can range from $5000\text{ K} \rightarrow 80,000\text{ K}$

DA - most common

- broad H lines only in spectrum

DB - no H lines, only He

DC - no lines at all!

— almost all C & O —

White dwarf cooling

There are no nuclear reactions in a white dwarf — it is a ball of inert C & O which may have a small H/He atmosphere



What is the source of its luminosity ?

→ Thermal energy $U = \frac{3}{2} N k T_c$

$\frac{\partial U}{\partial t}$ interior temp

We can derive luminosity from

$$L = - \frac{\partial U}{\partial t} \quad \text{and eq's of stellar structure}$$

Find that

$$L_{\text{wd}} = L_0 \left(1 + \frac{5}{2} \frac{t}{\tau_0} \right)^{-7/5}$$

τ_0 is cooling timescale $\propto T_0 \sim 2 \times 10^8 \text{ yr}$

We can use this to estimate
the age of clusters and of
the stellar disk (and compare with
stellar evolution ages)

As nuclei cool they eventually
settle into a solid :

'crystallization'

Electron degeneracy

- No 2 electrons can occupy exactly the same quantum state
(Pauli exclusion principle)
- Uncertainty principle
 $\Delta p \Delta x \gtrsim \hbar$
(can be thought of as defining available quantum states)

A classical (ideal) gas will have no motion at $T = 0$

A degenerate electron gas does

Very high density e^- gas:

- ⇒ Δx between neighboring electrons very small
- ⇒ Δp must be correspondingly large
- Electrons must have momenta that differ by $\gg \frac{h}{\Delta x}$ or Pauli exclusion principle is violated (strictly, can have 2 because of spin)
- So some electrons have very high momenta
- This leads to a high pressure which is independent of ~~does~~ temperature in the case of complete degeneracy.

$$\rho \propto P^{5/3} \quad (\text{non-relativistic } e^- \text{ degeneracy})$$

$$\& \rho \propto \rho T \quad (\text{ideal gas})$$

How dense do we need to be?

white dwarfs have density $\sim 10^6 \text{ g/cm}^3$

at 10^7 K , degeneracy pressure is
 $\sim 100 \times$ thermal pressure!

Mass-radius relation

The more massive a white dwarf, the smaller it is:

$$\text{Hydrostatic } \equiv^n \frac{dP}{dr} = -\rho g r$$

$$\text{one shell approx. } \frac{P_c}{R} = \frac{m}{\frac{4}{3}\pi R^3} \cdot \frac{GM}{R^2}$$

$$P_c \simeq \frac{GM^2}{R^4}$$

$$\text{Now } \rho \propto \rho^{5/3} \propto \left(\frac{m}{R^3}\right)^{5/3}$$

$$\text{So } \left(\frac{m}{R^3}\right)^{5/3} \propto \frac{gm^2}{R^4}$$

$$\frac{m^{5/3}}{R^5} \propto \frac{gm^2}{R^4}$$

$$\Rightarrow R \propto m^{-1/3}$$

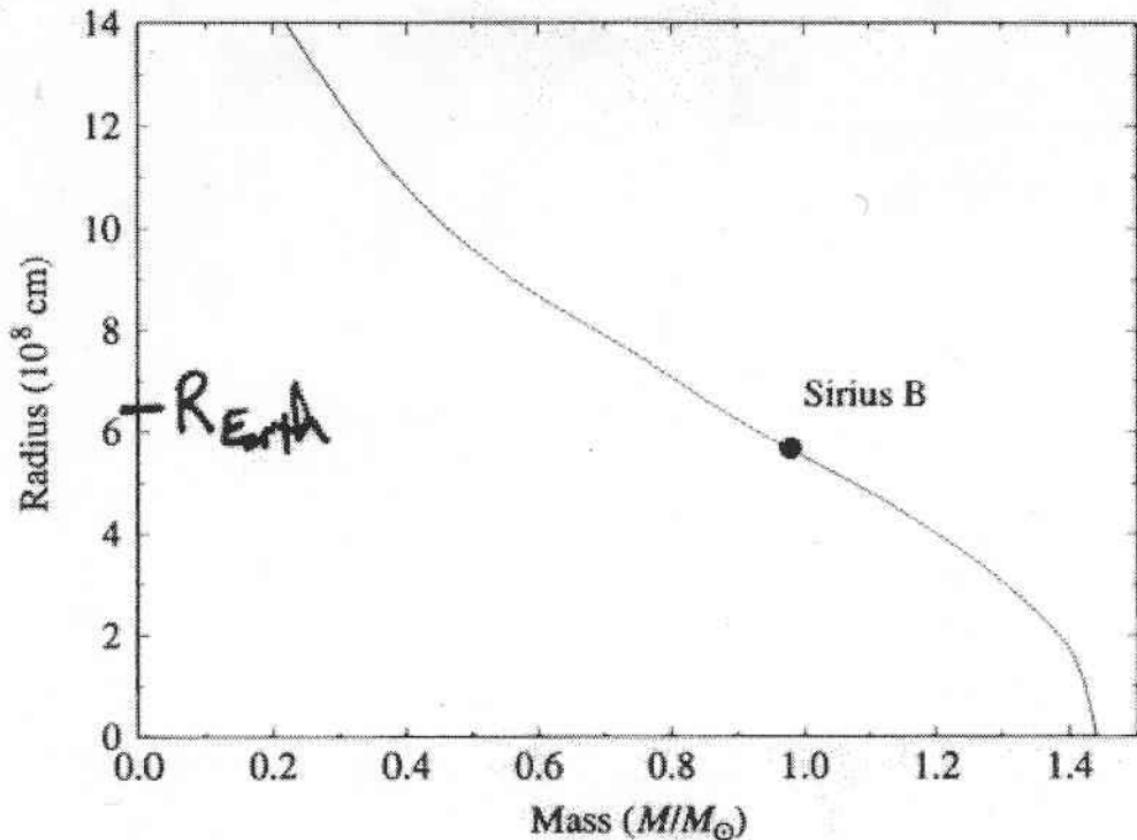


Figure 15.7 Radii of white dwarfs of $M_{\text{wd}} \leq M_{Ch}$ at $T = 0$ K.

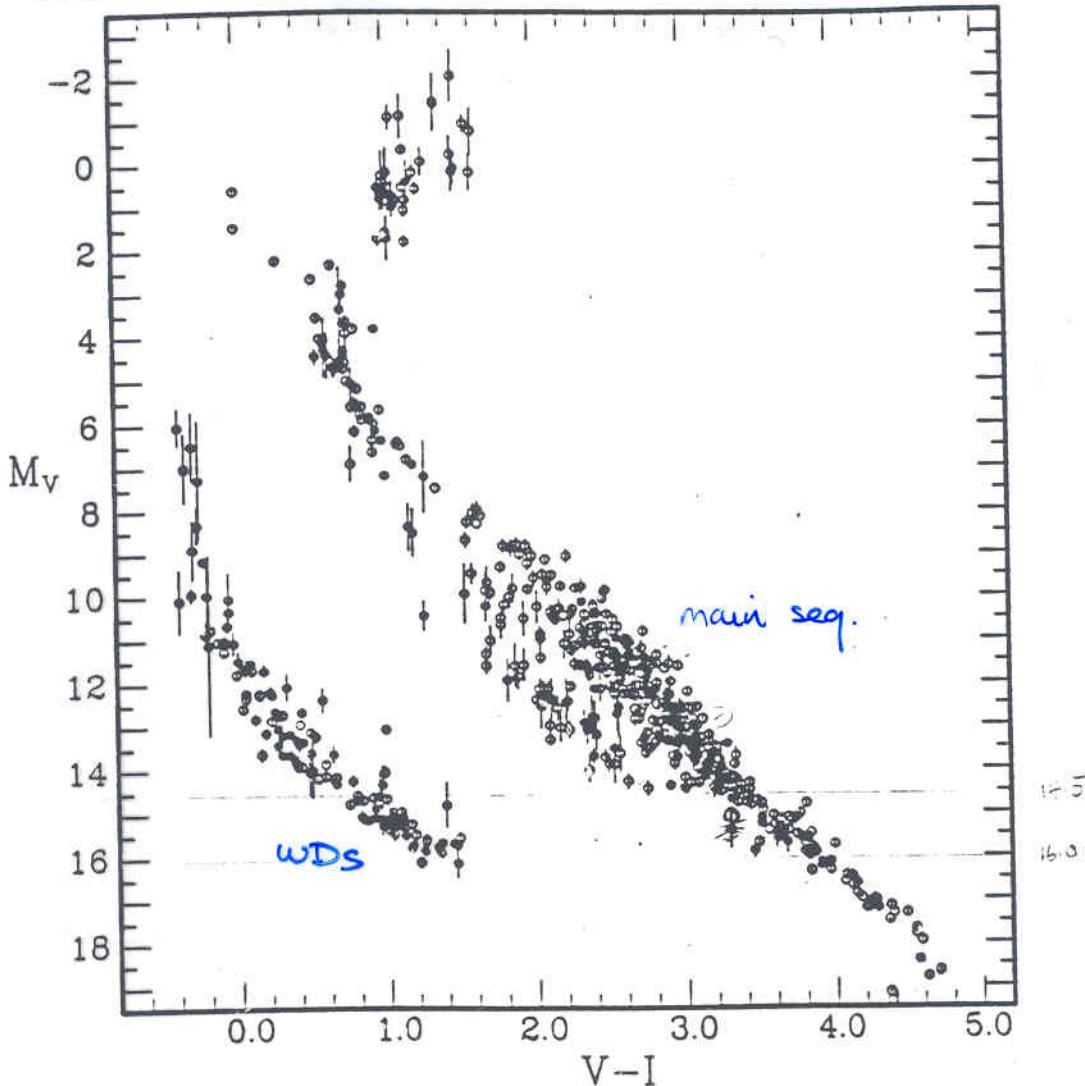


Fig. 1.1. The M_V vs $V-I$ color-magnitude diagram for a selection of 680 stars with (a) published USNO parallaxes (open circles; see text) or (b) unpublished but completed or nearly completed USNO parallaxes (filled circles).

Fig. 1.1 shows the M_V vs $V-I$ color-magnitude diagram for a selection of 510 stars with published USNO parallaxes along with 170 of the completed or nearly completed stars. The stars with published parallaxes in the figure were selected for meeting the following criteria: formal mean errors in M_V of a) $\leq \pm 0.20$ for the dwarfs, b) $\leq \pm 0.40$ for the subdwarfs, and c) $\leq \pm 0.15$ for the degenerates. (Giants and subgiants are shown, irrespective of the parallax quality, due to the small numbers of such stars.) All points are plotted with ± 1 m.e. error bars but, due to the scale of the diagram, they often do not show.

There is a limit to the mass that
 e^- degeneracy pressure can support
against gravity - WHY ?

Chandrasekhar limit

We can estimate the speed of electrons
in a white dwarf like Sirius B :
(see C&O for calculation)

$$v \approx 1.1 \times 10^{10} \text{ cm/s}$$

ie $\frac{1}{3}$ speed of light !

If star is more massive, density \uparrow
and $v \uparrow$

Finite speed of light imposes limit
on mass of a white dwarf

Chandrasekhar limit $M = 1.44 M_{\odot}$

Q Imagine a white dwarf star
close to the Chandrasekhar limit
in a close binary system, accreting
mass from its companion. What
might happen?

This is one theory for the origin of
a Type I supernova.