

Fusion reactions

The first nuclear reactions made on Earth were fission ones : splitting apart large nuclei



Why is it so difficult to make a fusion reaction work on Earth ?

→ Coulomb barrier

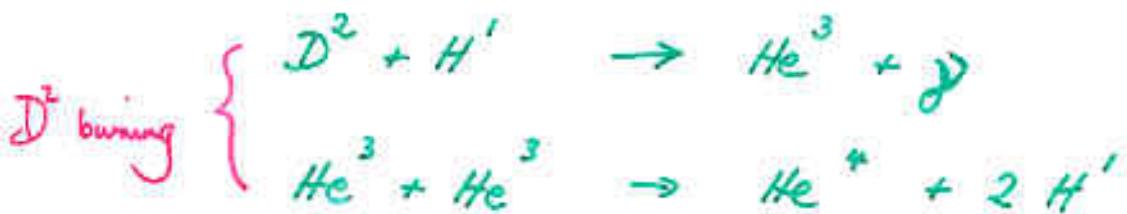
Nucleosynthesis

Fusion reactions

(a) in Big Bang, Universe was very hot and dense, and some light elements (up to Li^7) were produced. Measurements of primordial abundances of these elements, especially He^4 , show very good agreement with predictions – strong confirmation of Big Bang

(b) anything heavier than Li was made in stars.

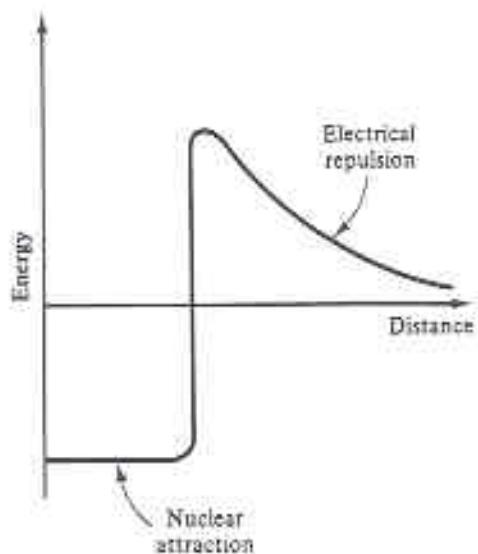
Proton-proton chain:



Need very high temperatures to overcome Coulomb barrier

THE FUSION BARRIER

Figure 9.3 The potential energy for two protons as a function of distance. This includes the electrical force and a model for the nuclear force.



To combine two protons, they need to have sufficient k.e. to get close enough to overcome repulsive force

Maxwell-Boltzmann distⁿ: particle energies distributed like $e^{-E/kT}$

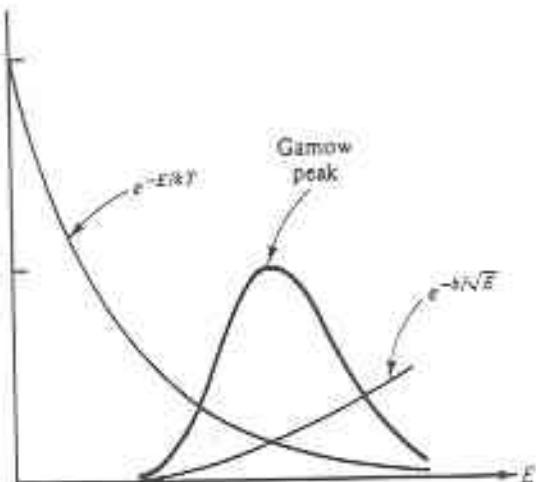
Quantum-mechanical tunneling: there is a finite probability that a particle will be closer to other one than classical physics would predict

Probability ~~takes~~ of penetration

$$\propto e^{-1/\sqrt{E}}$$

k.e.: the faster the better
tunneling: the slower the better

Figure 9.4 The probability of a nuclear fusion, as a function of the particle energy E , at a given gas temperature. This shows the combined effects of the location of the classical turning point and quantum-mechanical tunneling.



PROTON-PROTON CHAIN



weak interaction (neutrino produced)
goes very slowly, needs $T \sim 10^7 K$



(more quickly)



(most common)

In total : 4 protons in

1 ${}^4\text{He}$ nucleus out, plus

2 positrons, 2 γ rays, 2 neutrinos

Table 5-1 Reactions of the PP chains

Reaction	Average			$\frac{dS}{dE}$	B	τ_{12} , years†
	Q value, Mev	ν loss, Mev	S_0 , kev barns			
$H^1(p, \beta^+ \nu) D^2$	1.442	0.263	3.78×10^{-22}	4.2×10^{-24}	33.81	7.9×10^9 ←
$D^2(p, \gamma) He^3$	5.493		2.5×10^{-4}	7.9×10^{-6}	37.21	4.4×10^{-8}
$He^3(He^3, 2p) He^4$	12.859		5.0×10^3		122.77	2.4×10^5
$He^3(\alpha, \gamma) Be^7$	1.586		4.7×10^{-1}	-2.8×10^{-4}	122.28	9.7×10^5
$Be^7(e^-, \nu) Li^7$	0.861	0.80				3.9×10^{-1}
$Li^7(p, \alpha) He^4$	17.347		1.2×10^2		84.73	1.8×10^{-5}
$Be^7(p, \gamma) B^8$	0.135		4.0×10^{-2}		102.65	6.6×10^1
$B^8(\beta^+ \nu) Be^{8*}(\alpha) He^4$						3×10^{-8}
	18.074	7.2				

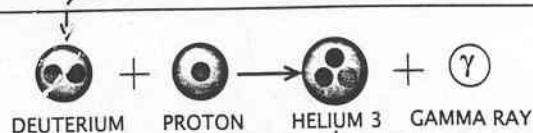
† Computed for $X = Y = 0.5$, $\rho = 100$, $T_6 = 15$ (sun).

PP REACTION



BUT ONE TIME IN 230:

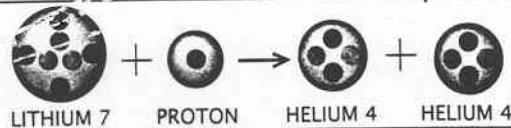
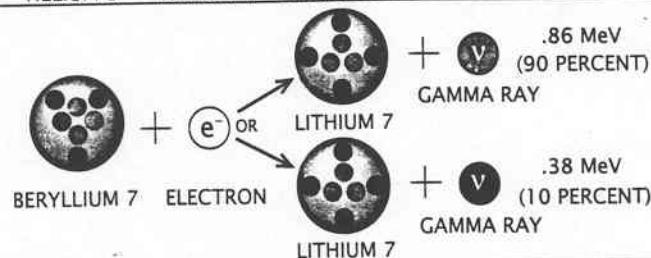
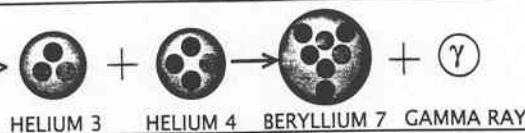
PEP REACTION



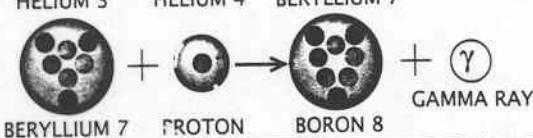
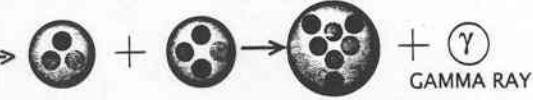
BRANCH 1
(85 PERCENT)



BRANCH 2
(15 PERCENT)

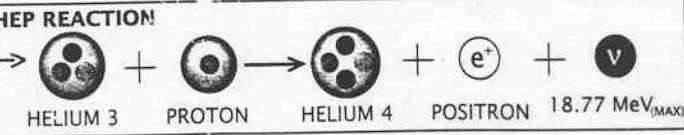


BRANCH 3
(.02 PERCENT)



HEP REACTION

BRANCH 4
(.00002
PERCENT)



^{37}Cl expt needs

• 81 MeV ν

" ^{75}Ge needs .23 MeV ν
only

PROTON-PROTON CHAIN of nuclear reactions is thought to provide more than 98 percent of the sun's energy. The initial proton-proton reaction produces the vast majority of solar neutrinos; these neutrinos have too little energy to be observed in the chlorine detector but should show up in the new gallium detectors. Energetic boron 8 neutrinos are thought to dominate the solar neutrinos detected in the chlorine experiment. The even more energetic helium-proton (HEP) neutrinos are so rare that they make only a minor contribution to the measured flux. The relative frequency of these reactions is based on inferences regarding conditions in the solar interior; if the inferences are wrong, the neutrino-flux predictions also could be wrong.

Proton-proton chain is basic energy source of Sun.

Nuclear fusion will continue until ~~the~~ the Sun's core has been converted to ${}^4\text{He}$.

Energy released by converting 4 protons to one ${}^4\text{He}$ atom:

Since $E = mc^2$, can work this out by comparing masses

$$m_p = 1.6726 \times 10^{-24} \text{ g}$$

$$\text{mass of } {}^4\text{He nucleus} = 6.6464 \times 10^{-24} \text{ g}$$

$$4m_p = 6.6904 \times 10^{-24} \text{ g}$$

$$4m_p - m({}^4\text{He}) = 0.007 m_p$$

(missing is 51, hence 1)

Assume most of Sun is protons, and 10% will participate in fusion

available mass = $0.1 M_{\odot}$

available energy = $0.007 m_p c^2$ per proton

$$\begin{aligned}\text{Total energy} &= 0.007 \times 2 \times 10^{32} \times (3 \times 10^{10})^2 \\ &= 1.3 \times 10^{51} \text{ ergs}\end{aligned}$$

Problem : what is the lifetime of the Sun with this energy production ?

(c) In massive stars with temperature $>$ Sun's, another fusion process, needing ^{12}C as catalyst, operates to convert $\text{H} \rightarrow \text{He}$:

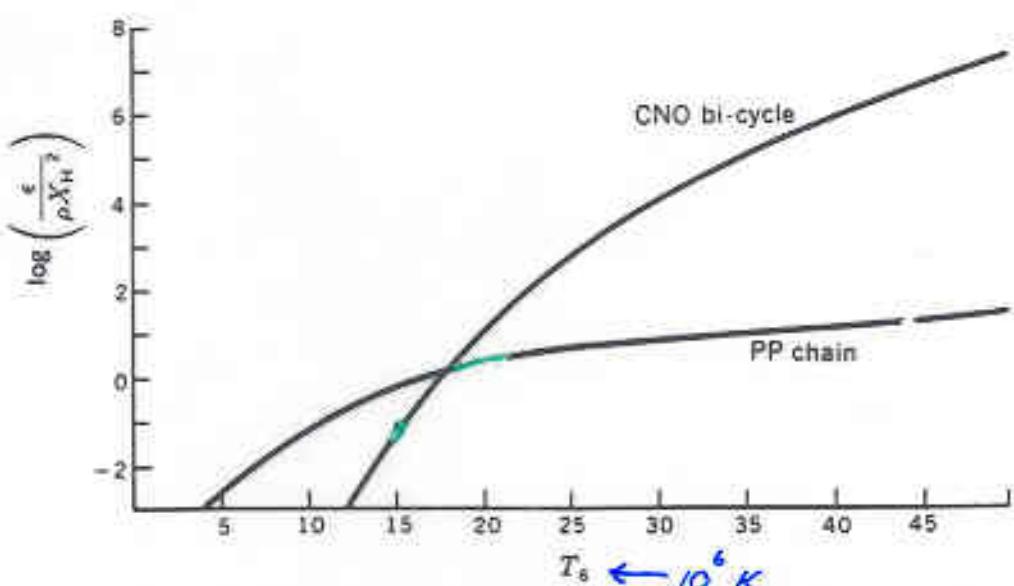


Fig. 5-16 A comparison of thermonuclear power from the PP chains and the CNO cycle. Both chains are assumed to be operating in equilibrium. The calculation was made for the choice $X_{\text{CN}}/X_{\text{H}} = 0.02$, which is representative of population I composition.

OTHER FUSION REACTIONS

(need hotter temperatures than core of Sun)

TRIPLE ALPHA PROCESS



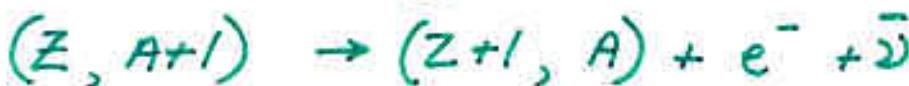
This needs temperature $\sim 10^8 \text{ K}$

NEUTRON CAPTURE

For high atomic number nuclei, electrical repulsion too high for fusion with charged particles. Use neutrons.



- slow w.r.t. beta decay:



"s-process"

NUCLEAR BINDING ENERGIES

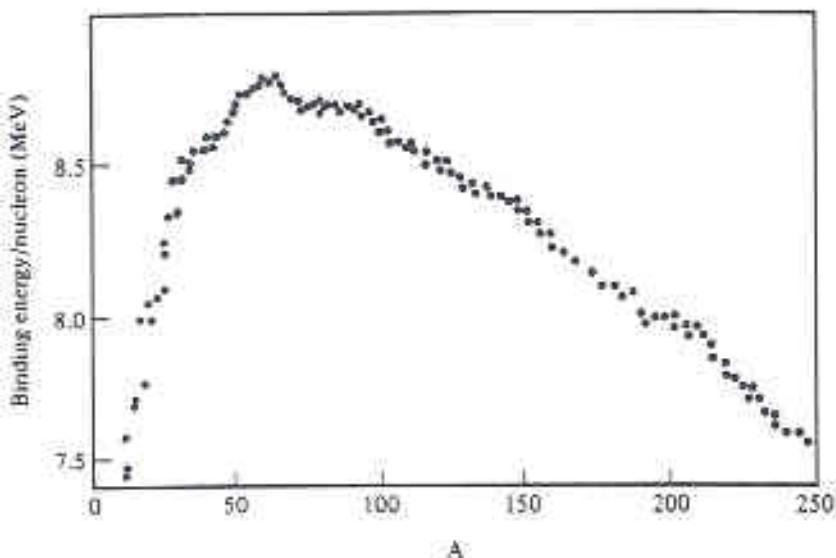


Figure 9.2 Nuclear binding energies. The horizontal axis is the mass number A , and the vertical axis is the total binding energy of the nucleus, divided by the number of nucleons in the nucleus. Nuclei with a higher binding energy per nucleon are the most stable.

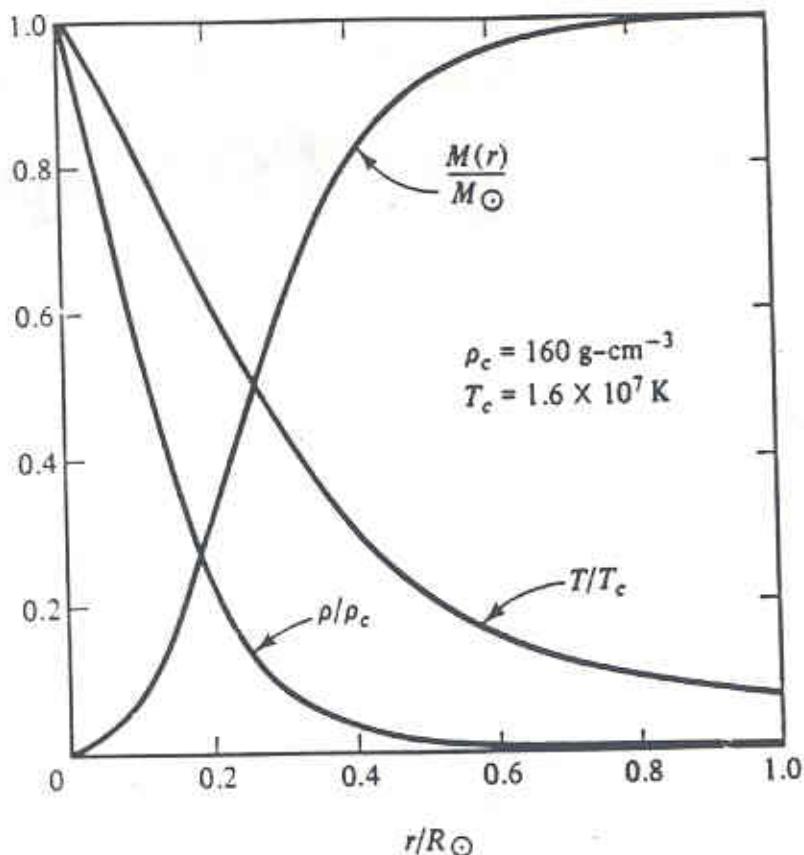
Binding energy of nucleus : work required to disassemble it.

Higher binding energy \Rightarrow a more stable nucleus.

isotope

Question : which/has the most stable nucleus ?

Stellar models



- Energy generation from fusion reaction calculations
- Energy transport — radiation or convection
- Hydrostatic equilibrium (pressure vs gravity)
- Equation of state ρ as $f(P, T)$

+ large computer

→ Stellar model

* structure determined by mass, chemical composition, age *

Q

We can measure many of the nuclear reaction rates, estimate temperature & density at surface of Sun, & get a model for the Sun's structure & energy output.

Give ≥ 3 ways we could test the model

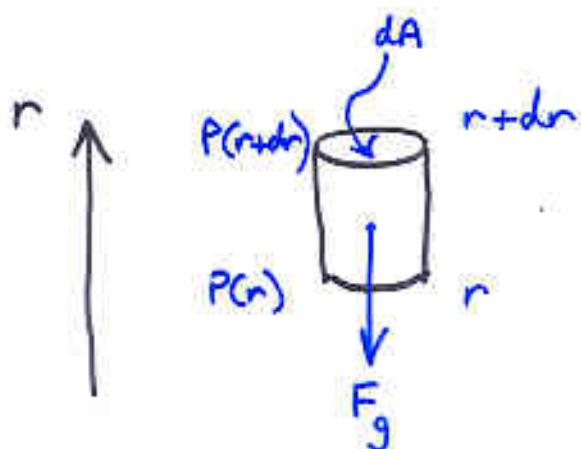
- Sun's photospheric properties
- Helioseismology
- Solar neutrinos
- lifetimes of Sun-like stars

HYDROSTATIC EQUILIBRIUM

(नियन्त्रण)

The Sun is balanced between :

- collapsing under its own gravity , &
- expanding due to outward pressure of hot gas .



Consider a volume element of gas.

Ideal gas law $\rho = \frac{R}{\mu} \rho T$

ρ pressure , T temperature

ρ density , μ molecular weight , R gas constant

Equation of state for ideal gas ρ, p, T

$$\rho = \left(\frac{\rho}{m}\right) kT$$

m is mass per particle

use this & estimate of central pressure to get an estimate of the central temp. of the Sun

H completely ionized $\Rightarrow m = \frac{1}{2} m_p$

$$T_c = \frac{m P_c}{\rho k}$$

$$= \frac{\frac{1}{2} m_p \cdot P_c}{\rho k} \cdot \frac{\frac{4}{3} \pi R^3}{M_0} \cdot \frac{1}{k}$$

$$= 4.4 \times 10^7 \text{ K}$$

Equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho(r)g(r)$$

Use this eqn to estimate the central pressure of the Sun by considering whole Sun as one shell

$$\Delta P = P_c \quad (\rho = 0 \text{ at edge})$$

$$\Delta R = R$$

$$\frac{\Delta P}{\Delta R} = -\frac{GM}{R^2} \cdot g$$

$$g \text{ is average density } \approx \frac{M}{R^3}$$

$$\Delta P = P_c = \frac{GM}{R^2} \cdot g \cdot R$$

$$\approx \frac{GM}{R} \cdot \frac{M}{R^3} = \frac{GM^2}{R^4}$$

substituting for $G, M \& R$ we get $P_c \approx 10^{16} \text{ dyn/cm}^2$

- Downward force F_g = mass \times accelⁿ from gravity
 $= dm \times g_r$
 $= \rho dr dA \cdot g_r$
- Difference in pressure is
 $P(r+dr) - P(r)$
- Force due to this is $[P(r+dr) - P(r)] dA$

Balance forces : gravity vs pressure

$$-\rho dr dA \cdot g_r = (P(r+dr) - P(r)) dA$$

$$\text{So } -\rho g_r dr = P(r+dr) - P(r)$$

$$\frac{P(r+dr) - P(r)}{dr} = -\rho g_r$$

$$\boxed{\frac{dP}{dr} = -\rho g_r}$$

eqn of hydrostatic \equiv^n .

Q.

What are some everyday examples
of hydrostatic equilibrium ?

Q

Work out why the Sun doesn't
explode , but just keeps burning at
a roughly constant rate for billions
of years