

Measuring stellar mass

(and radius

and temperature)

Not only a basic quantity for understanding
structure & evolution of stars, but useful
for planet searches.

Visual binaries - can see both stars
(nearly!)

Position vector \underline{R} : mass weighted
average of position

$$\underline{R} = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2}$$

This is zero in center of mass frame

$$\frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2} = 0$$

$$\text{So } \underline{r}_1 = \frac{-m_2}{m_1 + m_2} \underline{r} \quad \& \quad \underline{r}_2 = \frac{m_1}{m_1 + m_2} \underline{r}$$

$$\text{if } \underline{r} = \underline{r}_2 - \underline{r}_1$$

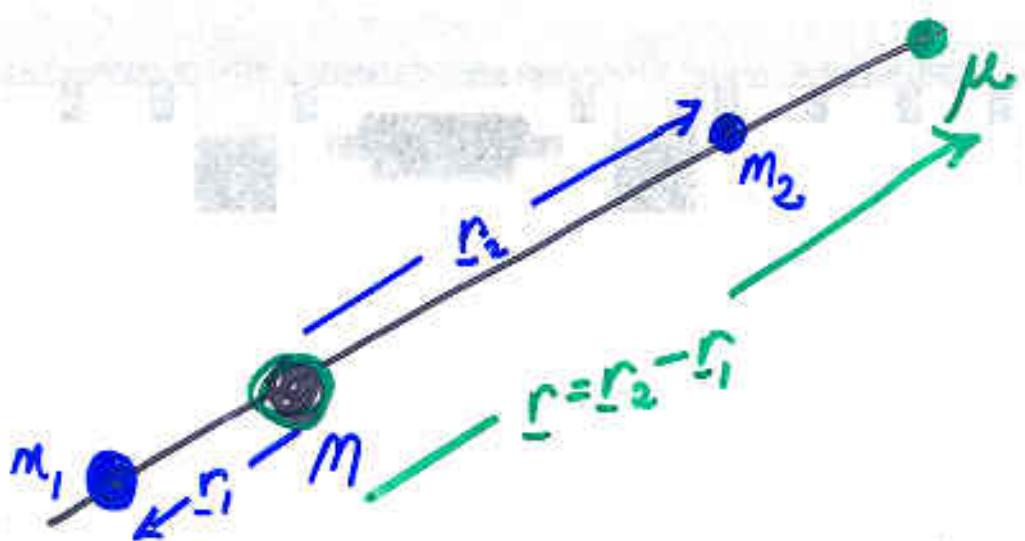
same distance to both stars so if α_i is angular distance from c.o.m. to star 1, etc

$$\frac{\underline{r}_2}{\underline{r}_1} = \frac{m_1}{m_2} \quad \& \quad \boxed{\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}}$$

We can derive the ratio of masses from the distances of the stars from their center of mass.

$$\text{Reduced mass } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Two-body problem reduces to mass μ orbiting mass $M = m_1 + m_2$



Semi-major axis a of μ 's orbit about M is sum of semi-major axes a_1 & a_2 of m_1 & m_2 . (approximate)

Kepler's 3rd law becomes :

$$\rho^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

We have m_1/m_2 & $m_1 + m_2$ so can derive each mass separately

But we need a expressed as a linear distance , not an angle so need to know the distance to the star

Q What does a typical parallax distance error do ^{to} the mass estimate ?

→ There are few visual binaries known and few good parallax estimates , so good estimates of stellar mass are rare .

Eclipsing binaries

If binary system is far enough away that the components can't be separated visually , the spectrum can still give information about the velocities of each component .

For stars too distant to be resolved as visual binaries, we can use velocity information if spectral lines are visible from both stars.

For circular orbits velocity is constant in magnitude : (not direction!)

$$v = \frac{2\pi a}{P}$$

So ratio of masses can be found from velocities :

$$\frac{m_2}{m_1} = \frac{v_1}{v_2}$$

If binary system is not exactly edge-on, what would velocity curve look like?

What if orbits were elliptical?

In general we don't know inclination of system, which is a problem

BUT

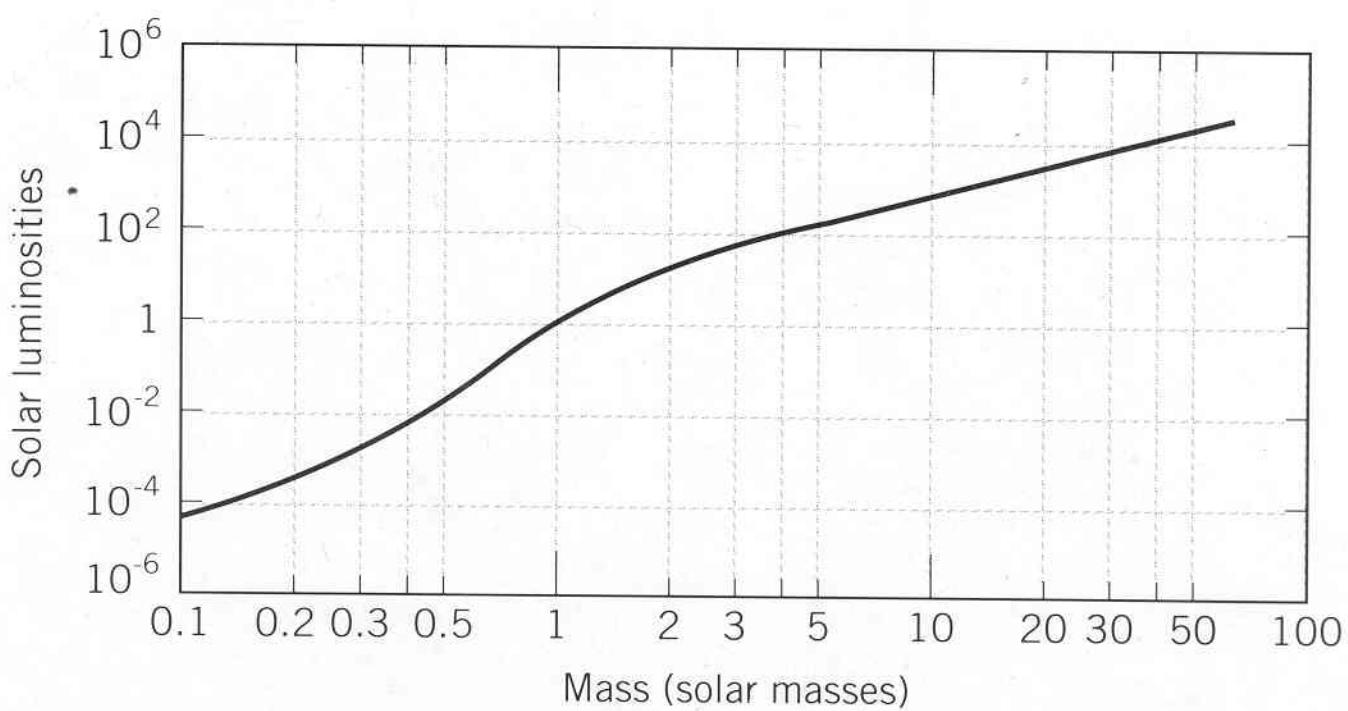
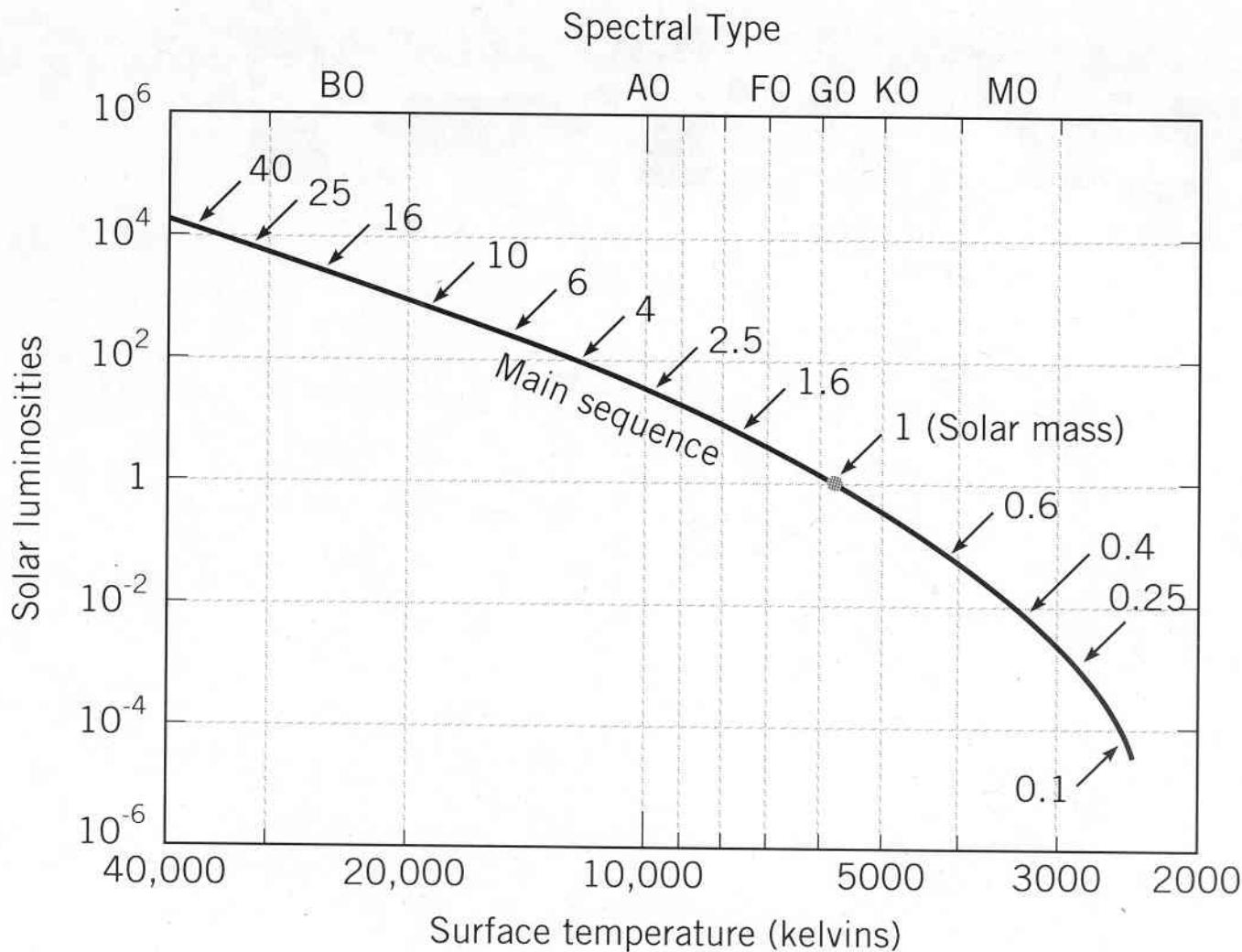
if the binary is an eclipsing system we know it must be very close to edge-on.

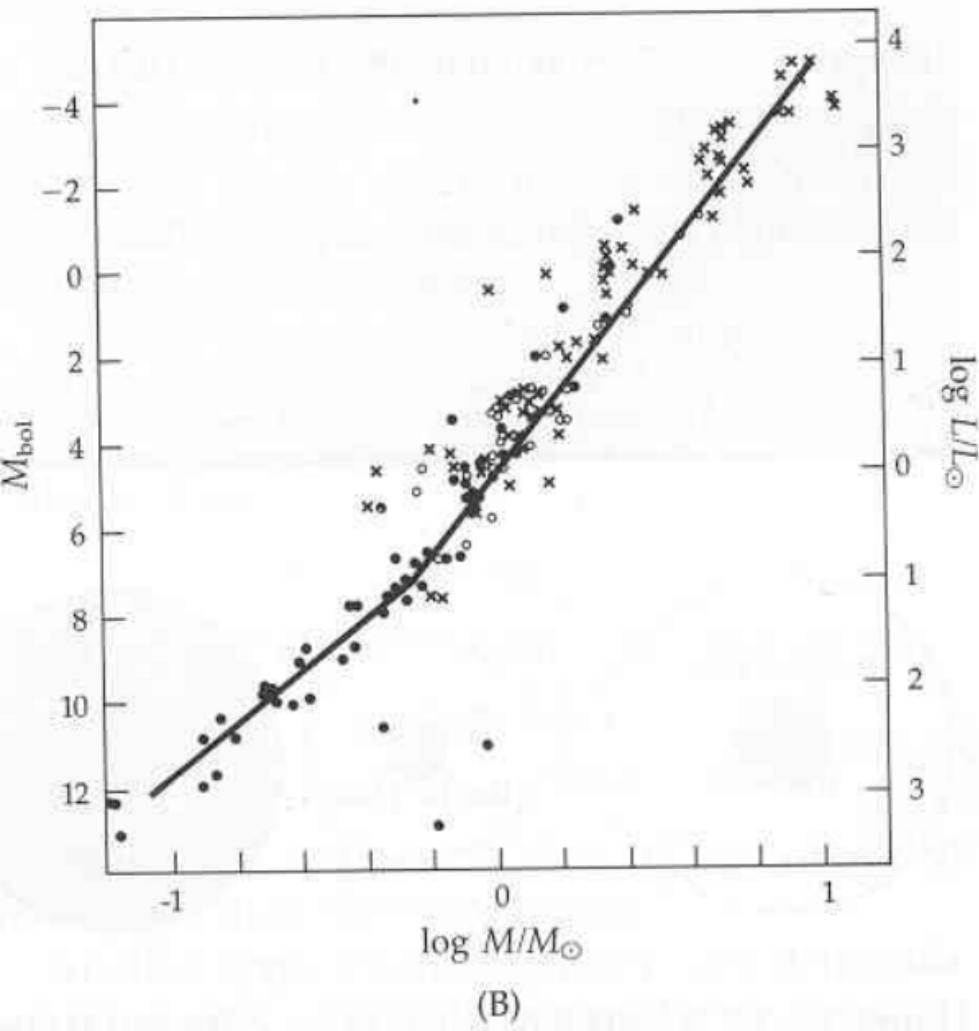
Can then use reduced-mass frame & Kepler's 3rd law:

$$m_1 + m_2 = \frac{P}{2\pi G} (v_1 + v_2)^3$$

Can also measure radii of stars from accurate light curves

$$\rightarrow T_{\text{eff}} \quad \text{since } F_{\text{surf}} = \sigma T_{\text{eff}}^4$$





Stellar sizes

$$\text{recall } L = \sigma T^4 4\pi R^2$$

If we have distance & magnitude m , can get L .

To get T calibrated, need R

- eclipsing binaries
- lunar occultations
- speckle interferometry & AO

Figure 5.15 Stellar sizes determined from eclipsing binaries. The orbit is shown above and the light curve is shown below. The lengths of the eclipses, and the steepness of the sections at the beginning and end of each eclipse (such as A-B and E-D) depend on the sizes of the stars.

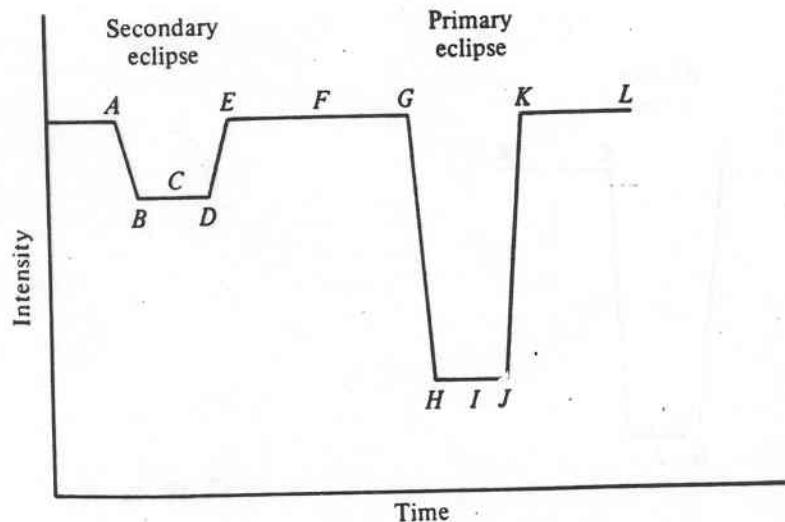
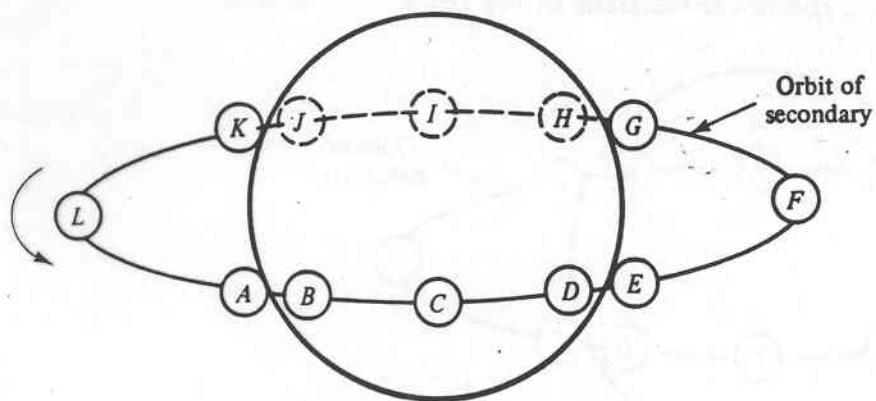
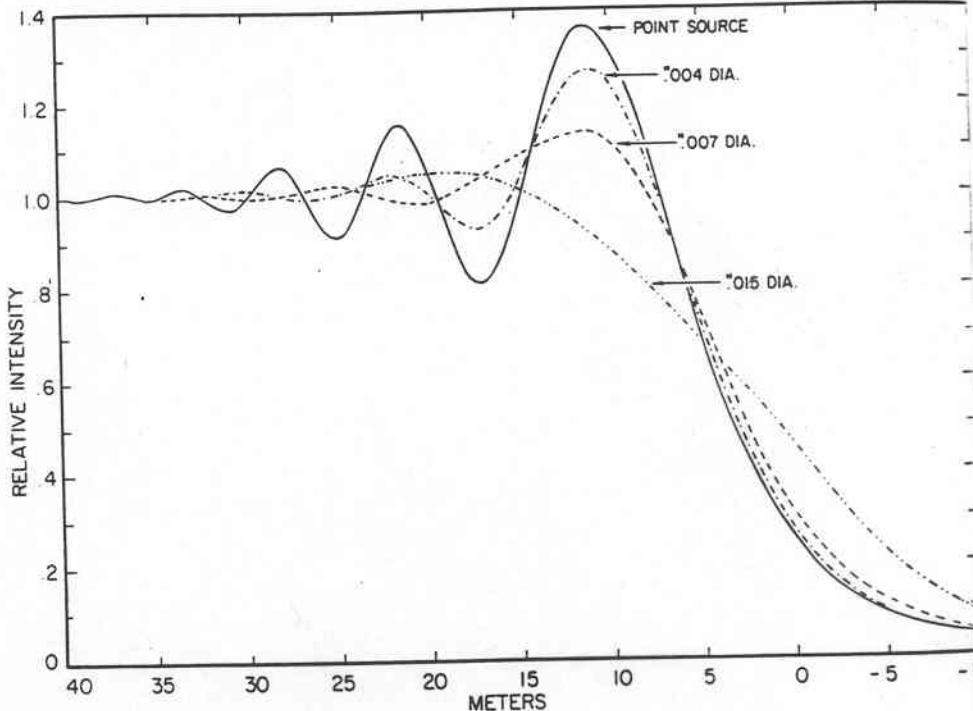
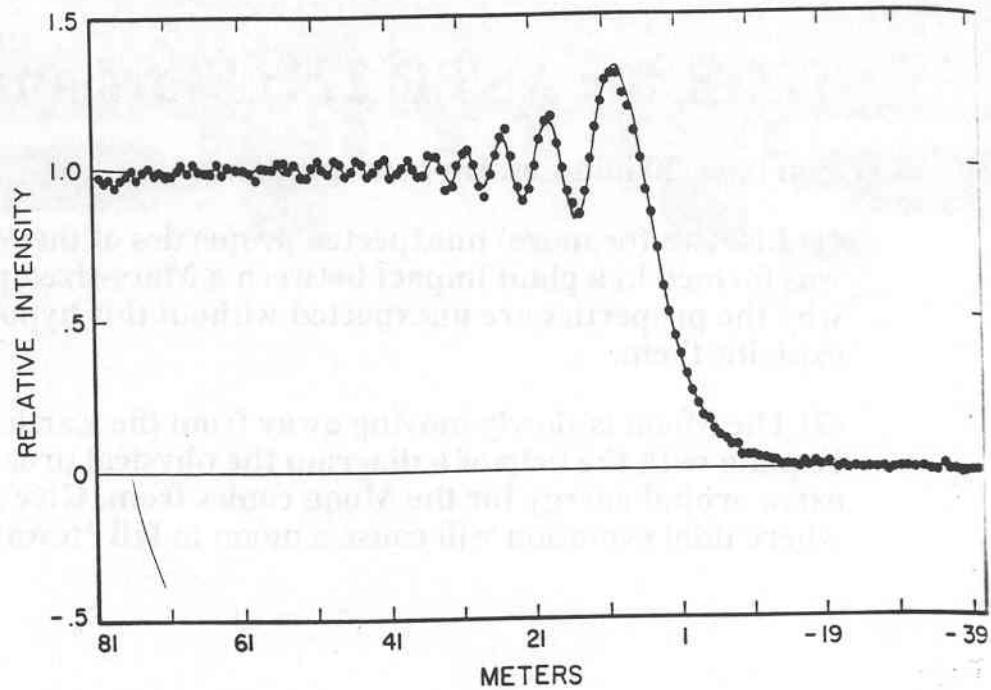


Figure 5.16 (a) Tracing of a lunar occultation of the star β Sco A. On the left we see the star before the occultation, and on the right it is occulted. The wiggles in the curve are due to diffraction effects as the star passes behind the lunar limb. The dots are the actual data, and the smooth curve is the best fit of a theoretical model to the data. (b) Theoretical calculations of what the curves in (a) would look like for stars of different angular sizes.



The stellar luminosity function

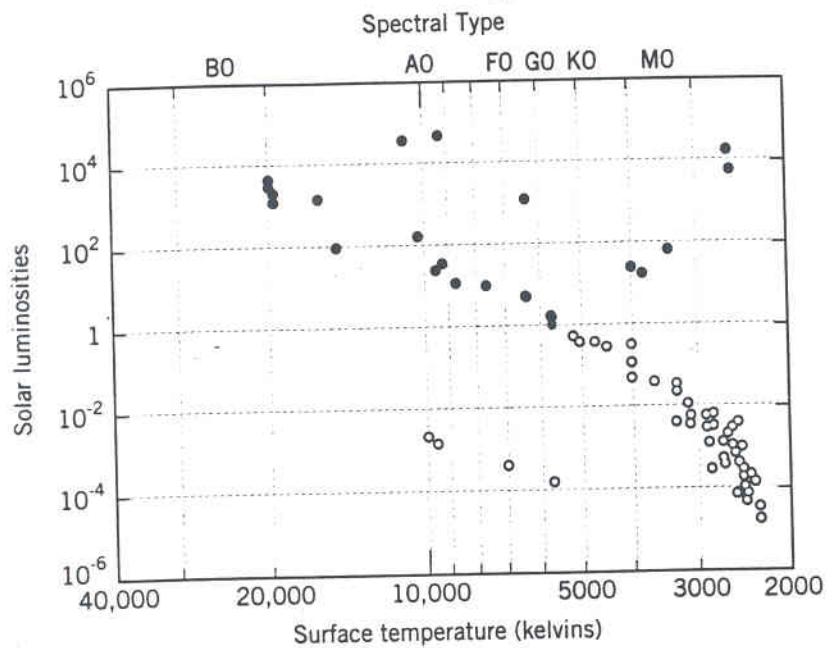


FIGURE 15–7. The data from Figures 15–5 and 15–6 have been plotted together in this diagram. Note the separation into distinct groups.

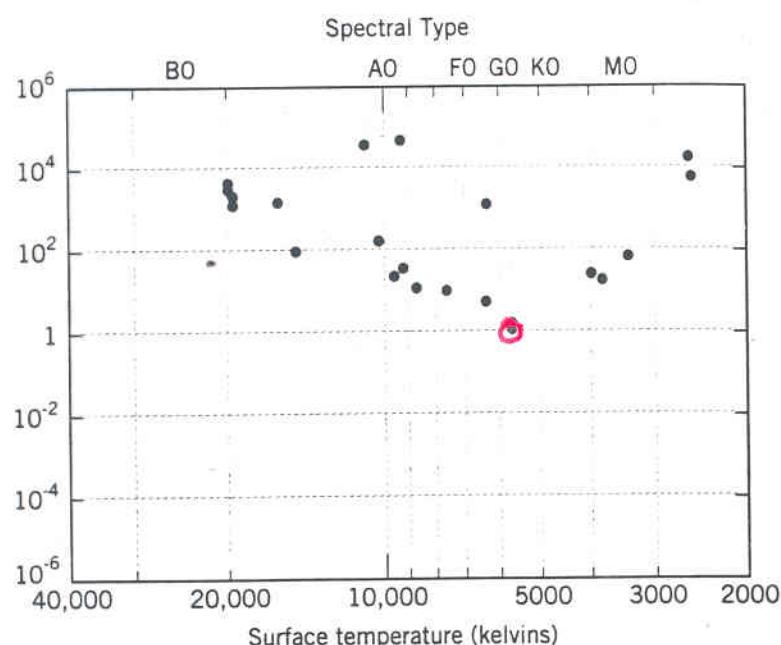


FIGURE 15–6. The Hertzsprung-Russell diagram for the rightmost stars in the night sky. The Sun is denoted as an orange point.

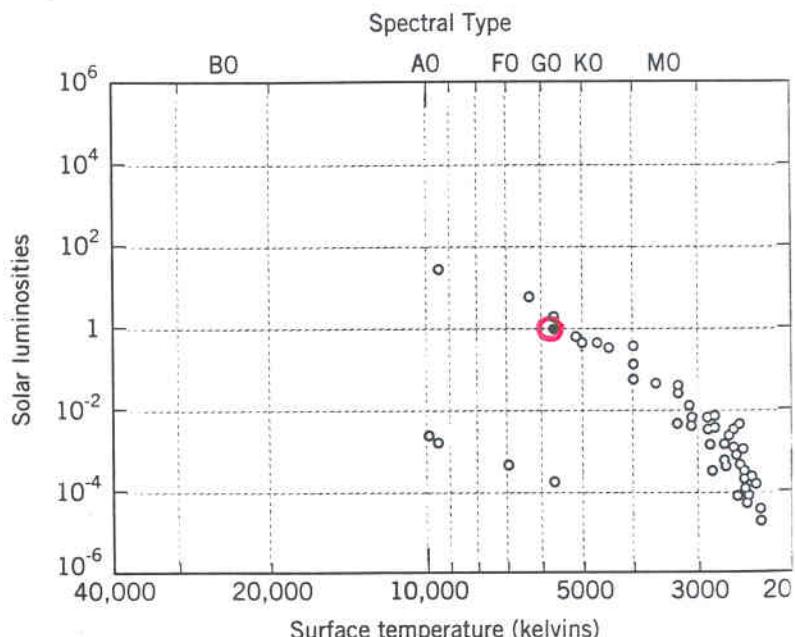


FIGURE 15–5. The Hertzsprung-Russell diagram for the stars closest to the Sun. The Sun is denoted as an orange point.

Stellar luminosity function

- The least luminous stars are the most common ones

(makes sense : the nearest known star, Proxima Centauri, has $M_V = 15$)

- This is partially initial conditions
 - more low mass stars are formed ; and partially due to stellar lifetimes — high mass stars evolve much faster

THE SUN

Radius & Temperature

- Mass $M_0 = 1.99 \times 10^{33}$ g.
..... 99.9 % of mass of solar system
- Radius $R_0 = 6.96 \times 10^8$ cm
- Average density 1.4 g/cm^3

The Sun is an 'average' star :

stellar masses range from $0.1 M_0$

to (?) $100 M_0$

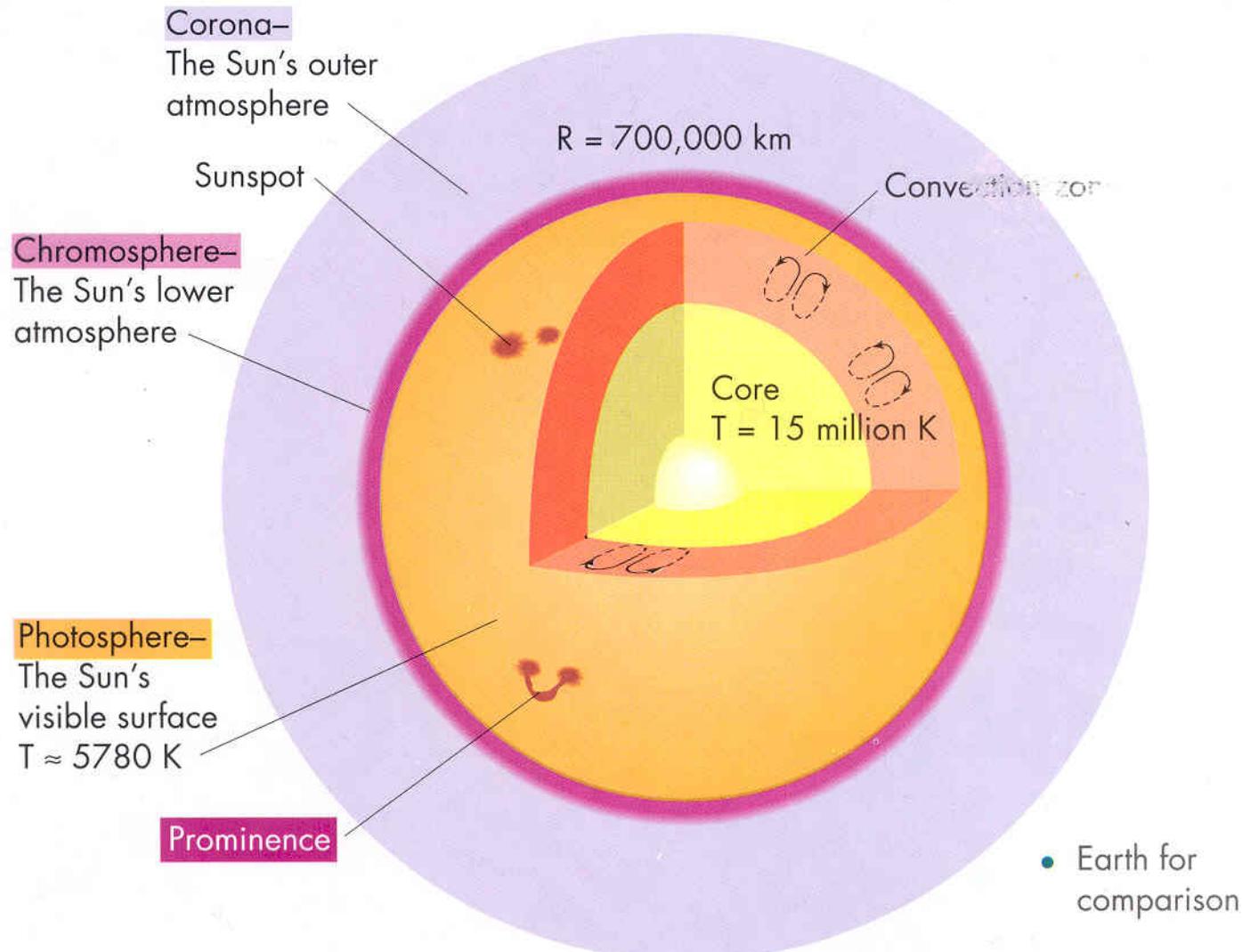
'average' evolutionary state.

- Luminosity : total energy / sec given off by Sun

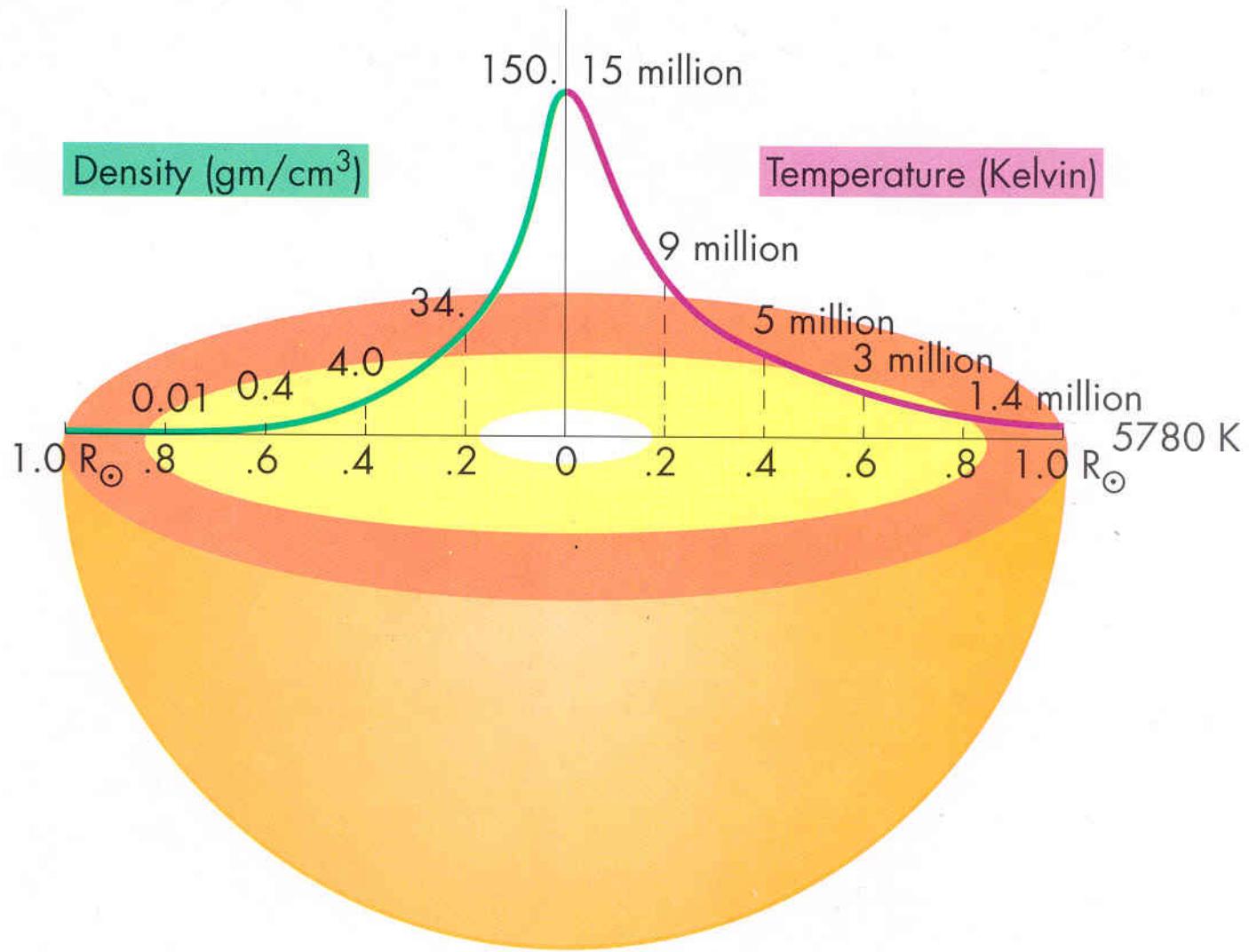
$$L_0 = 3.83 \times 10^{33} \text{ ergs/sec}$$

- Temperature at photosphere (surface)
 $5800 K$

The Sun's interior and atmosphere (Fig. 10-1)



Density and temperature of the Sun (Fig. 10-2)



The Sun as an 'average' star

HR diagram shows there are stars that are

- $10^5 \times$ more luminous
- 100 \times more massive
- 4 \times hotter

AND

- 10^4 times less luminous
- 10 times less massive
- 6 times cooler

RADIATIVE TRANSFER

How does the energy get out?

Nuclear reactions give off energy in core as γ -rays.

Through most of Sun's interior, these are absorbed and re-emitted by deuterium atoms, and are degraded to lower energy in the process.

This is a slow process, called a "random walk" because the emitted photon may move in any direction

9.3 Radiative Transfer

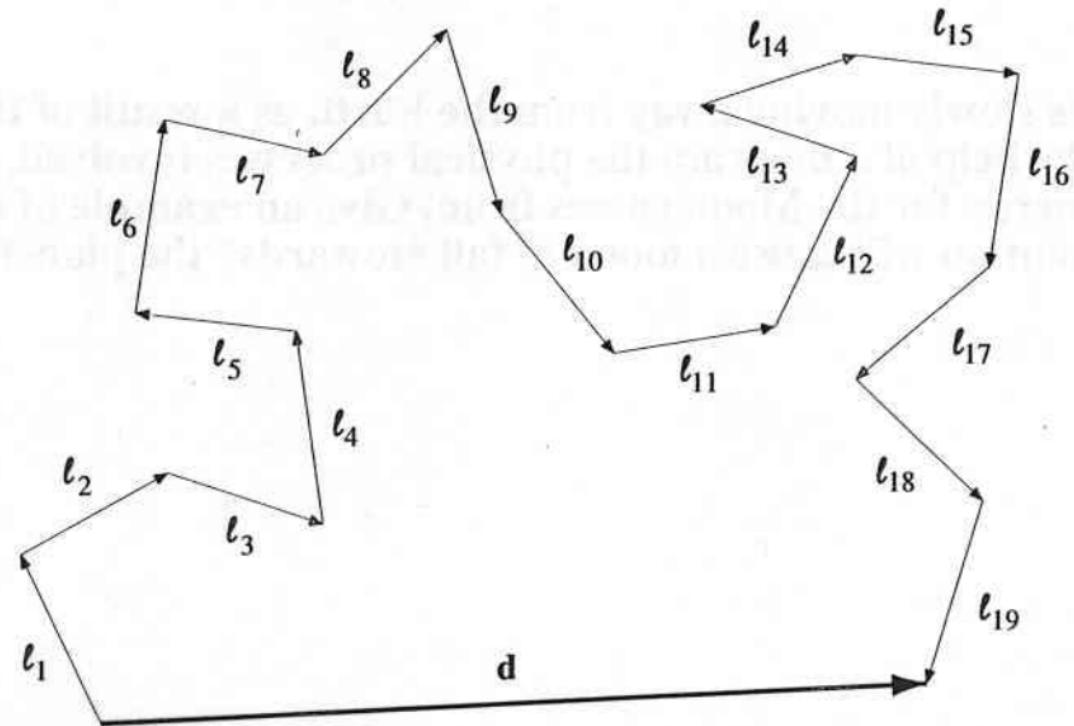
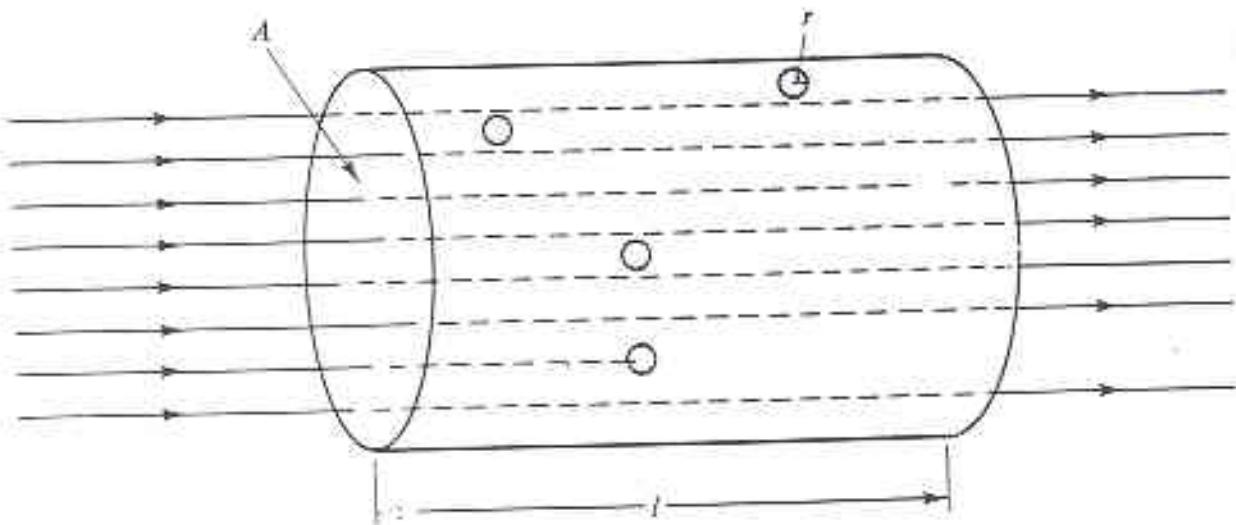


Figure 9.11 Displacement \mathbf{d} of a random-walking photon.

Optical depth

Transfer of radiation through an absorbing medium (atmosphere of star, Earth's atmosphere)

Simple model: absorbers are spheres, radius r



Cross-section for absorption σ

In this case, we put $\sigma = \text{cross-sectional area of sphere}$

$$\sigma = \pi r^2$$

(σ is used in a more probabilistic sense in quantum mechanics, nuclear physics, etc.)

Consider a cylinder length ℓ

~~area~~ area A

n spheres / unit volume

$$\text{Volume of cylinder} = \ell A$$

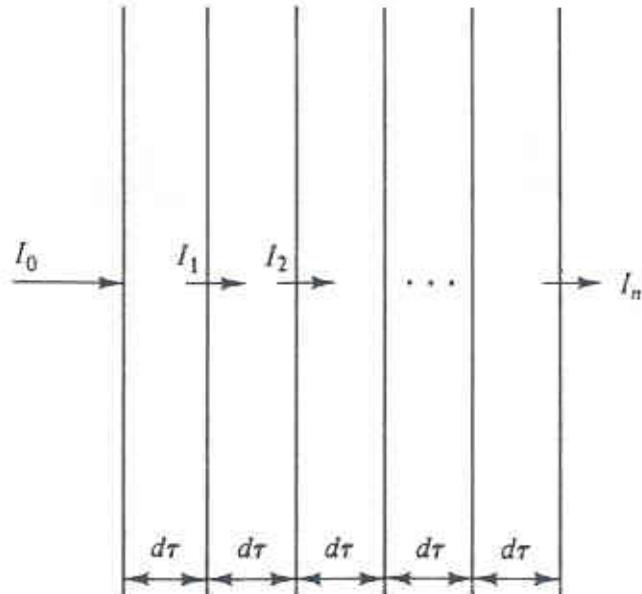
$$\text{Total no of spheres } N = n \ell A$$

IF spheres don't shadow each other
(ie $\sigma_{\text{tot}} \ll A$)

$$\begin{aligned}\text{Total cross-sectional area seen by incoming}\\ \text{radiation } \sigma_{\text{tot}} &= N \sigma \\ &= n \ell A \sigma\end{aligned}$$

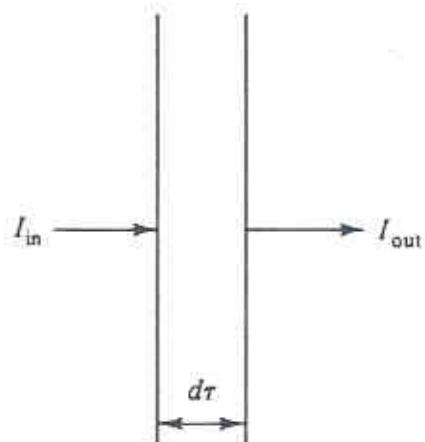
$$\begin{aligned}\text{Fraction of light absorbed is fraction}\\ \text{covered by spheres} &= \frac{\sigma_{\text{tot}}}{A} \\ &= \frac{n \ell A \sigma}{A} \\ &= n \ell \sigma\end{aligned}$$

This was derived
assuming few
absorptions



$$(\delta_{tot} \ll A \\ \text{ie } nCA\delta \ll A)$$

$$nC\delta \ll 1 \\ \gamma \ll 1 \quad)$$



What if you have many?

Divide into small layers with $\gamma \ll 1$ for each

$$dI = I_{out} - I_{in}$$

$$= -I d\gamma \quad \gamma \text{ is fraction absorbed}$$

IF $\gamma \ll 1$

$$\frac{dI}{I} = -d\gamma \quad \text{negative since intensity } \downarrow$$

Integrate this : γ' goes from 0 to γ
 I' goes from I_0 to I

$$\frac{dI'}{I'} = - d\gamma'$$

$$\int_{I_0}^I \frac{dI'}{I'} = - \int_0^\gamma d\gamma'$$

$$[\ln I']_{I_0}^I = - [\gamma']_0^\gamma$$

$$\ln \frac{I}{I_0} = - \gamma$$

$$I = I_0 e^{-\gamma}$$

So for more absorptions, γ is not fraction absorbed - radiation coming out goes exponentially with γ

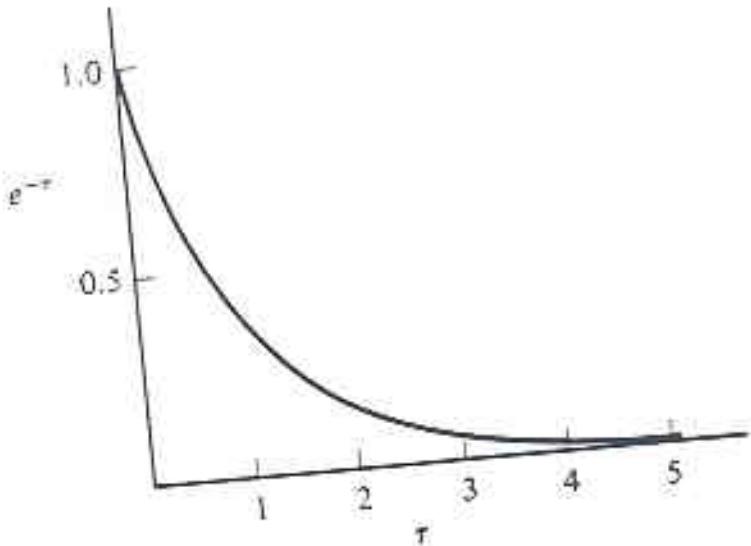


Figure 6.5 $e^{-\tau}$ vs. τ , showing the falloff in transmitted radiation as the optical depth increases. Note that the curve looks almost linear for small τ . For large τ , it asymptotically approaches zero.

For $\gamma \ll 1$ $e^{-\gamma} \approx 1-\gamma *$

$$\text{So } I = I_0 (1-\gamma)$$

and γ is fraction absorbed

— Maximum absorbed has to be 1! —

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- Absorption coefficient $\kappa_\lambda = n\sigma_\lambda$
 (fraction)
 number of absorptions per unit length,
 since it is κ_λ/c
- Mean free path = length between absorptions
 $= \frac{1}{\kappa_\lambda}$
 $= \frac{1}{n\sigma_\lambda}$

In a typical gas at room temperature & pressure
 molecules are of size $\sim 10^{-8} \text{ cm}$ (1\AA)
 mean free path (for collisions) $\sim 10^{-5} \text{ cm}$
 ie $1000 \times$ diameter of molecule

In the real case of radiative transfer through the Sun, we need to consider emission as well as absorption in each layer.

In general

- "Optically thin" $\tau \ll 1$ (most light gets thru)
- "Optically thick" $\tau \gg 1$ (most light absorbed)

Examples of optically thick:

Earth's atmosphere in far-UV

~~interior~~ interior of Sun

Energy flow from the Sun's core to
its surface (Fig. 10-3)

