Galaxy formation: cosmological context

(if you have done ASTR 328/428, review notes on the growth of structure)

At recombination (where we see Cosmic Background Radiation),
the universe was hot and very smooth → no longer smooth \( \nabla^2 z = 0 \)

But there were slight density variations
( amplified from quantum fluctuations by inflation)
They grow gravitationally over time
→ how they grow depends on cosmological model

To treat this mathematically, assume homogeneity/isotropy
gravity (no \( \Lambda \))
(see Mihos 328/428 notes for more
longer 'Galaxy formation' ch 11 detail)

matter & radiation treated as fluid with density \( \rho \), velocity \( \vec{v} \)

\[
\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v}
\]
continuity equation

\[
\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \nabla \rho - \nabla \phi
\]
equation of motion

\[
\nabla^2 \phi = 4\pi G \rho
\]
Poisson's equation
We can then perturb the equations \((\frac{\delta p}{p} \rightarrow r(1+\delta))\) and watch how the perturbation \(\delta\) grows as universe expands.

Growth equation \(\ddot{\delta} + 2 \frac{\dot{R}}{R} \dot{\delta} = 4\pi G \rho \delta\) \((R\) is scale factor of universe\).

Two special cases:

(i) flat universe \(S_0 = 1\)

\[
\frac{\delta p}{p} = \delta \times t^{2/3} \times \frac{1}{1+z}
\]

So, in flat universe, perturbations keep growing.

(ii) empty universe \(S_0 = 0\) (if there were only baryons, no dark matter, no dark energy)

\[
S(t) = A + Bt^{-1}
\]

\(A\) stays constant, \(B\) decays.

* If we only had baryons in our universe, the initial small perturbations would not grow into galaxies.*

\(\delta_0 = 0.05\)
What about cosmology with $\Lambda$ (preferred today)?

$$S_m + S_{\Lambda} = 1 \quad (\text{flat})$$

- Early on, universe behaves like $S_m = 1$
  - So perturbations grow like $t^{2/3}$

- Things change around $z = \frac{1}{S_0}$ ($= 3$ for $S_0 = 3$)

- Perturbations grow faster than if $S = 0$ (good!)
  - But in this case, more slowly than $S_m = 1$

- Things slow down when acceleration becomes important ($z \approx 0.5, S_0 = 3$)
Fig. 11.4. The growth of density perturbations over the range of scale-factors $R = 10^{-3}$ to 1 for world models with $\Omega_A = 0$ and density parameters $\Omega_0 = 0.01, 0.1, 0.3$ and 1.
Fig. 11.5. The growth of density perturbations over the range of scale-factors $R = 1/30$ to 1 for world models with $\Omega_0 + \Omega_\Lambda = 0$ and density parameters $\Omega_0 = 0.1$, 0.3 and 1.
Early evolution: linear regime

We can treat the slightly over-dense region as its own model universe:

![Fig. 11.1](image)

Fig. 11.1. Illustrating a spherical perturbation with slightly greater density than the average in a uniformly expanding Universe. The region with slightly greater density behaves dynamically exactly like a model Universe with density \( \rho_0 + \delta \rho \).

Surrounding universe is flat, so will keep expanding.

Over-dense region is closed, so will expand and then recollapse. Density grows linearly with scale factor \( R \).

![Fig. 11.2](image)

Fig. 11.2. Illustrating the growth of a spherical perturbation in the expanding Universe as the divergence between two Friedman models with slightly different densities.

Densest initial fluctuations will collapse first; more modest early decreasing will take longer to recollapse.
The distribution and kinematics of early high-\( \sigma \) peaks in present-day haloes: implications for rare objects and old stellar populations

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Abstract

We show that the hierarchical assembly of cold dark matter haloes preserves the memory of the initial conditions. Using N-body cosmological simulations, we demonstrate that the present-day spatial distribution and kinematics of objects that formed within early \( (z \gtrsim 10) \) protogalactic systems (old stars, satellite galaxies, globular clusters, massive black holes, etc.) depends mostly on the rarity of the peak of the primordial density field to which they originally belonged. Only for objects forming at lower redshifts does the exact formation site within the progenitor halo (e.g. whether near the centre or in an extended disc) become important. In present-day haloes, material from the rarer early peaks is more centrally concentrated and falls off more steeply with radius compared to the overall mass distribution, has a lower velocity dispersion, moves on more radial orbits, and has a more elongated shape. Population II stars that formed within protogalactic haloes collapsing from \( \sim 2.5(J_{\text{fluctuations}}) \) would follow today an \( r^{-3.5} \) density profile with a half-light radius of 17 kpc and a velocity anisotropy that increases from isotropic in the inner regions to nearly radial at the halo edge. This agrees well with the radial velocity dispersion profile of Galaxy halo stars from the recent work of Battaglia et al. and with the anisotropic orbits of nearby halo stars.

Key words: methods: N-body simulations – Galaxy: halo – Galaxy: kinematics and dynamics – galaxies: formation – galaxies: haloes – galaxies: star clusters.

1 INTRODUCTION

In a universe where cold dark matter (CDM) dominates structure formation, the haloes of galaxies and clusters are assembled via the hierarchical merging and accretion of smaller progenitors (e.g. Lacey & Cole 1993). This process causes structures to relax violently to a new equilibrium by redistributing energy among the collisionless mass components. Early stars formed in these progenitors behave as a collisionless system just like the dark matter particles in their host haloes, and they undergo the same dynamical processes during subsequent mergers and the buildup of larger systems like massive galaxies or clusters. It is of crucial importance in galaxy formation studies to explore the efficiency of the mixing process and to see if any spatial or kinematical signatures exist in material that collapses at different epochs and within peaks of the primordial Gaussian density field of different rarity.

In this paper, we use a suite of high-resolution cosmological N-body simulations to analyse the distribution and kinematics within present-day galaxy haloes of dark matter particles that originally belonged to selected branches of the merger tree. These properties are particularly relevant for the baryonic tracers of early CDM structures, e.g. the old stellar halo which may have originated from the disruption at high redshift of numerous dwarf protogalaxies (Bullock, Kravtsov & Weinberg 2000), the old halo globular clusters, and also giant ellipticals (Gao et al. 2004). The end product of the entire merger tree is a triaxial cuspy dark matter halo (Dubinski & Carlberg 1991; Navarro, Frenk & White 1996; Moore et al. 1999; Diemand et al. 2005): a small fraction of early progenitor systems survive the merging process and end up as dark matter substructures (Ghigna et al. 1998). Since rare, early haloes are strongly biased towards overdense regions (e.g. Cole & Kaiser 1989; Sheth & Tormen 1999), i.e. towards the centres of larger-scale fluctuations that have not collapsed yet, we might expect that material originating from the earliest branches of the merger tree is today much more centrally concentrated than the overall halo. Indeed, a 'non-linear' peak biasing has been discussed previously by several authors (Moore et al. 1998; White & Springel 2000; Moore 2001).

Here we show that the distribution and kinematics of 'old material' within present-day galaxy haloes depends primarily on the rareness of the peaks of the primordial density fluctuation field to which it originally belonged. Specifically, today’s properties of objects that formed in old rare density peaks above \( \sigma(M, z) \) depend largely on \( \sigma \) and not on the particular values of \( z \) and \( M \). [Here
given redshift. The redshifts of the simulation outputs and the halo masses corresponding to these fluctuations are given in Table 2. These values are for the cosmological model we have simulated \((\Omega_m, \Omega_{\Lambda}, \sigma_8, h) = (0.268, 0.732, 0.7, 0.71)\). In a \(\sigma_8 = 0.9\) the same fluctuations would collapse earlier (at \(z_{0.9}\), see Section 2.1).

3.1 Radial distribution

High-\(\sigma\) material is strongly biased towards the centre of present-day haloes (see upper panels of Figs 4-12), with the rarer peaks showing stronger bias. Figs 4-9 show different \((M, z)\) selections.
Table 1. Present-day properties of the six simulated dark matter haloes. The columns give halo name, spline softening length, number of particles within the virial radius, virial mass, virial radius, peak circular velocity, and radius to the peak of the circular velocity curve.

<table>
<thead>
<tr>
<th>Halo</th>
<th>ε0</th>
<th>N_{vir}</th>
<th>M_{vir}</th>
<th>r_{vir}</th>
<th>V_{c,max}</th>
<th>r_{Vc,max}</th>
</tr>
</thead>
<tbody>
<tr>
<td>D12</td>
<td>1.8</td>
<td>14.0</td>
<td>305</td>
<td>1743</td>
<td>958</td>
<td>645</td>
</tr>
<tr>
<td>G0</td>
<td>0.27</td>
<td>1.7</td>
<td>1.01</td>
<td>260</td>
<td>162</td>
<td>52.2</td>
</tr>
<tr>
<td>G1</td>
<td>0.27</td>
<td>1.9</td>
<td>1.12</td>
<td>268</td>
<td>162</td>
<td>51.3</td>
</tr>
<tr>
<td>G2</td>
<td>0.27</td>
<td>3.8</td>
<td>2.21</td>
<td>337</td>
<td>190</td>
<td>94.5</td>
</tr>
<tr>
<td>G3</td>
<td>0.27</td>
<td>2.6</td>
<td>1.54</td>
<td>299</td>
<td>180</td>
<td>45.1</td>
</tr>
<tr>
<td>G4</td>
<td>0.27</td>
<td>0.25</td>
<td>0.144</td>
<td>138</td>
<td>96.4</td>
<td>15.0</td>
</tr>
</tbody>
</table>

corresponding to the same value of v. While the shapes of these profiles are similar for a given v, the normalization (or traced mass fraction) generally grows with increasing mass threshold and lower redshifts. The density profiles are averaged over the four galaxy haloes of similar mass. The scatter from halo to halo in their total density profiles and that of their subsets is relatively small, but there are substantial halo-to-halo variations in the anisotropy parameter β(r), which we discuss in Section 3.2.

Mass densities are fitted with a general \( \alpha \beta \gamma \) profile that asymptotes to a central cusp \( \rho(r) \propto r^{-\gamma} \):

\[
\rho(r, v) = \frac{\rho_s}{(r/r_s)^\gamma \left[ 1 + (r/r_s)^\alpha \right]^{(\beta-\gamma)/\alpha}}.
\]  

For comparison the NFW (Navarro, Frenk & White 1996) profile has \((\alpha, \beta, \gamma) = (1, 3, 1)\), while the Moore et al. (1999) profile has \((\alpha, \beta, \gamma) = (1.5, 3, 1.5)\). We fix \(\alpha = 1\) and the inner slope to \(\gamma = 1.2\), which is the best-fitting slope for the D12 cluster when resolved at very high resolution (\(m_{DM} = 3.0 \times 10^5 M_\odot\)) (Diemand et al. 2005). We fit the entire dark halo using an outer slope of \(\beta = 3\) to determine the scale radius \(r_s = r_{vir}/c\), where \(c\) is the concentration. To approximate the high-\(\alpha\) subset profiles we use a smaller scale radius \(r_{s'} = r_{s}/f_{s'}\) (corresponding to a higher concentration \(c_{s'} = f_{s}c\)) and also a steeper outer slope \(\beta_{s'}\). The \(f_{s}\) and \(\beta_{s'} \) values used in the plots are calculated with simple empirical formulae that approximately parametrize the entire range of profiles, i.e. peaks above 1 to 4\(\sigma\) and haloes ranging from a low-concentration \((c = 4.5)\) cluster halo to a small, \(c = 17\) galaxy halo:

\[
r_{s'} = r_{s}/f_{s'}, \quad f_{s} = \exp(v/2), \quad \beta_{s'} = 3 + 0.26v^{1.6}.
\]  

The values for the 1 to 4\(\sigma\) peaks are given in Table 3, and the profiles are plotted in the upper left panels of Figs 4–12 with open

Figure 3. The present-day distribution and kinematics of dark matter particles selected at \(z = 10.5\), averaged over four Milky Way sized simulated haloes. We marked the 52 most massive groups found with FOF using different linking lengths: \(b = 0.164\) (dotted lines), \(b = 0.164/2\) (long-dashed lines) and \(b = 0.164/3\) (short-dashed lines). Particles selected at the same redshift with overdensities above 1000 have similar present-day distributions (dash-dotted lines). For comparison, we also plot the same quantities using all particles from the final structure (solid curves).
To test whether, at the present epoch, massive clusters contain a different fraction of high-\(\sigma\) material than field galaxies, we have used a lower-resolution N-body simulation (300 particles in a 90 Mpc box, for a resolution of \(10^{10} M_\odot\) in order to obtain a larger galaxy/cluster sample. We have selected 10 parent hosts at \(z = 0\) with FOF masses close to \(2 \times 10^{11}, 10^{12}, 10^{13}\) and \(10^{14} M_\odot\), for a total of 40 haloes. We have then marked FOF groups with more than 100 particles at redshifts 7.0, 4.3, 3.1 and 0.8, and determined the fractions of the virial mass of the parent belonging to a fixed progenitor mass/redshift (\(M, z\) pair). The values obtained are given in Table 4. The mass fractions are practically constant for all parents that host any of the selected progenitors. As we select parents of lower masses, the number of parents hosting rarer (higher-\(\sigma\)) progenitors drops from 10 to 0 within about a decade in mass.

4 SOME APPLICATIONS

We have shown that the final distribution and kinematics of dark matter particles selected from early branches of the merger tree are systematically different from those of the parent halo as a whole. These properties are also relevant for old stellar populations if these form predominantly in early low-mass progenitor haloes, as stars behave essentially as collisionless systems just like the dark matter particles in our simulations. In the following we briefly discuss a number of possible applications.

4.1 Remnants of the first stars

Numerical simulations performed in the context of hierarchical structure formation theories suggest that the first (Population III) stars may have formed out of metal-free gas in dark matter minihaloes of mass above \(6 \times 10^5 M_\odot\) (Abel, Bryan & Norman 2000; Bromm, Coppi & Larson 2002; Yoshida et al. 2003; Kuhlen & Madau 2005) condensing from rare high-\(\sigma\) peaks of the primordial density fluctuation field at \(z > 20\), and were probably very massive. Barring any fine tuning of the initial mass function (IMF) of Population III stars, intermediate-mass black holes (IMBHs) (with masses above the \(5-20 M_\odot\) range of known ‘stellar-mass’ holes) may be one of the inevitable end-products of the first episodes of pre-galactic star formation (Madau & Rees 2001).

Where do relic pre-galactic IMBHs lurk in present-day galaxy haloes? To shed some light on this question, we have populated our 3σ (3.5σ) simulated progenitors at \(z_{\text{ion}} = 17.9\) \((z_{\text{ion}} = 21.2)\) with one seed IMBH for every \(6 \times 10^5\) solar masses of halo material. As discussed by Volonteri, Haardt & Madau (2003), these IMBHs will undergo a variety of processes during the hierarchical buildup of larger and larger haloes, like gas accretion, binary hardening, black hole mergers and triple interactions. While we neglect all of these effects here, our dark matter simulations do correctly model the bias in the formation sites, the accretion into larger haloes, and the

![Figure 7. Shape profiles of high-\(\sigma\) subsets at \(z = 0\) averaged over four galaxy haloes. The shapes are plotted for 3.5σ groups with virial mass \(\geq 4.9 \times 10^7 M_\odot\) at \(z = 13.7\) (squares), 3σ \([\geq 5.9 \times 10^8 M_\odot\) at \(z = 13.7\) (long-dashed lines); and \(\geq 2.2 \times 10^8 M_\odot\) at \(z = 10.5\) (short-dashed lines) and 2σ material \([\geq 4.8 \times 10^7 M_\odot\) at \(z = 7.5\) (dotted lines)]. The axial ratios of the entire dark matter halo are plotted with solid lines.](image-url)
competing effects of dynamical friction and tidal stripping within larger potential wells, as these complicated dynamical processes are dominated by the dark haloes that host the black holes. Therefore, in our toy model, the distribution of 3 or 3.5σ material at $z = 0$ describes the properties of holes wandering through within today’s galaxy haloes. The predicted IMBH number density and mass density profiles are shown as circles in Figs 8 and 9; the former may be regarded as an upper limit since we have neglected black hole mergers, while the latter have been estimated assuming that these off-nuclear black holes have grown by accretion to a mean mass of $1.5 \times 10^4 M_\odot$, which is a rough estimate obtained from fig. 14 of Volonteri et al. (2003).

Depending on the IMF of Population III objects, some first-generation low-mass stars may have survived until today. Their number-density profile $n(r)$ within the Milky Way can again be read following the circles in Fig. 8, under the assumption that $N = 1$ metal-free star survives for every $6 \times 10^5$ solar masses of 3σ progenitor halo material at $z_{0.9} = 17.9$. It is easy then to scale up the predicted value of $n(r)$ if $N \gg 1$ such stars were to survive instead. On average, we find that about one-third of these remnants would lie today in the bulge, i.e. in the inner 3 kpc. This fraction fluctuates between 24 and 45 per cent in the four galaxies G0–G3. The density in the Solar neighbourhood is of the order of $\approx 0.1 N/kpc^{-3}$, three orders of magnitude lower than in the bulge. The number density of remnants would be lower and more concentrated towards the Galactic Centre if Population III stars only formed within rarer 3.5σ peaks (Fig. 9). On average, about 59 per cent of them would now lie within the bulge (range is 38–84 per cent) and the local number density would be only $\approx 0.02 N/kpc^{-3}$. Even for $N = 10–100$, this is an extremely small value, many orders of magnitude below the local number density of halo stars. The above results suggest that the very oldest stars and their remnants should be best searched for within the Milky Way bulge.

4.2 Stellar haloes

Material from $> 2.5\sigma$ peaks has today a density profile that is very similar to the stellar halo around the Milky Way (Moore et al. 2005). It contributes a few per cent of the total virial mass, and therefore contains enough baryons to build up a $10^9 M_\odot$ stellar halo with a reasonable star formation efficiency. The assumption of a common pre-galactic origin between such a stellar component and the surviving Local Group dwarf galaxies provides an additional constraint and allows us to determine the progenitor mass threshold/redshift pair that best fits the data. From this argument Moore et al. (2005) identified hosts above $10^8 M_\odot$ at $z = 12$ as the progenitor haloes to which the bulk of halo stars originally belonged. To check whether this simple model reproduces the kinematics as well as the radial distribution of halo stars, we have compared the predicted radial velocity dispersion profile with recent data

Figure 8. Distribution of peaks above 3σ in present-day galaxy-sized haloes. Line styles are as in Fig. 4. The circles in the upper left panel show the estimated mass and number density of wandering IMBHs assuming an $N = 1$ seed hole for every $6 \times 10^5$ solar masses of 3σ progenitor halo material at $z_{0.9} = 17.9$, and a final black hole mass of $1.5 \times 10^4 M_\odot$ (see text for details).
Biasing

We have seen that it is easier for initially dense regions of the universe to collapse early. This is a result about dark matter, since this dominates the mass.

Galaxies (or even stars) may not form in all dark halos; they may only form at the densest peaks.

$$\text{Biasing} : \left( \frac{S_p}{\rho} \right)_{\text{galaxies}} = b \left( \frac{S_p}{\rho} \right)_{\text{dark matter}}$$
Figure 2. Density map of the high-resolution region of run G at $z = 0$. The particles marked in green were selected at $z = 13.7$ in groups above $4.9 \times 10^7 M_\odot$ (see Fig. 1). The squares enclose the virial radii of the five galaxy haloes analysed (G0 to G4).