ORBITS & KEPLER'S LAWS

Kepler's laws : planets around Sun, moons around planets, comets, binary stars

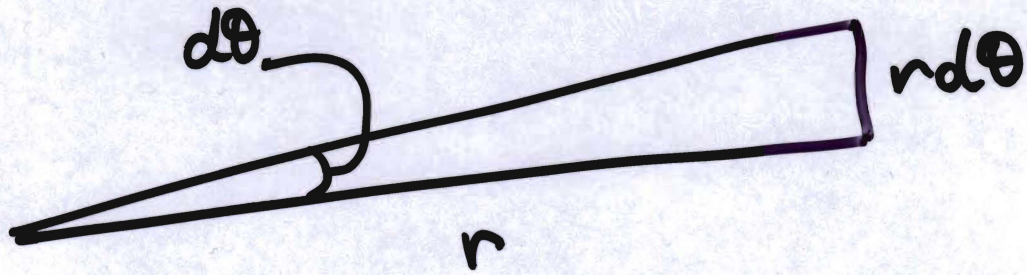
① The planets move in elliptical orbits with the Sun at one focus

② A line from the Sun to a planet sweeps out equal areas in equal times

$\boxed{Q}$ Prove Kepler's second law, using conservation of angular momentum

Hint : what area is swept out in time dt ?

In time dt, planet moves an angle $d\theta$ along its orbit.



Area of wedge is $\frac{1}{2} r \cdot r d\theta$

$\cancel{\frac{1}{2} r}$

Distance moved $= v_\theta \, dt = r d\theta$

So area $= \frac{1}{2} r \cdot v_\theta \, dt$

Angular momentum $\vec{L} = m \vec{r} \times \vec{v}$

$L = m r v_\theta$

Conservation of angular momentum

$\Rightarrow$ at any point $m_1 r_1 v_{\theta 1} = m_2 r_2 v_{\theta 2}$

Planet's mass unchanged, so

$$r_1 v_{\theta 1} = r_2 v_{\theta 2}$$

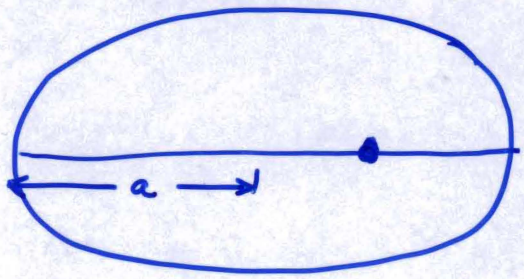
In time t:

$$\text{area} = \int_0^{t} \frac{1}{2} r v_\theta \, dt$$

Since $r v_\theta$ doesnt change along the orbit, neither does the area.

(3) The square of the period is proportional to the cube of the semi-major axis

$$P^2 \propto a^3$$



semi-major axis is a

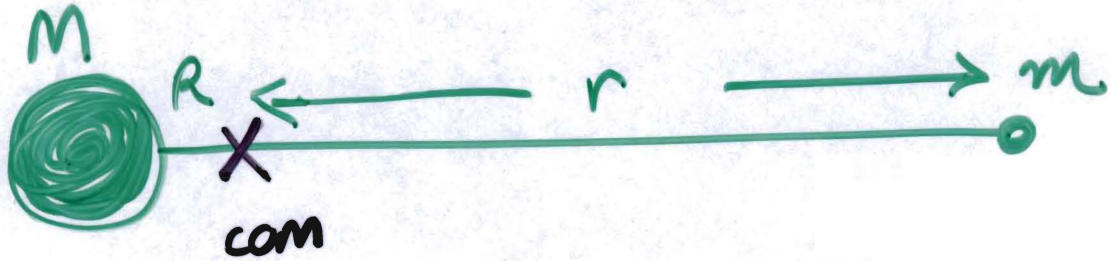
Prove for circular orbits by taking center-of-mass frame.

By defenition, Sun and planet stay on opposite sides of center-of-mass and their angular velocities about it are equal

Center of mass

The Earth and the Sun orbit around their center of mass.

Putting the center of mass at the origin:



$$MR = -mr$$

So a more correct statement of Kepler's first law is:

Each planet moves on an elliptical orbit with the center of mass at one focus.

Q

The Sun has mass 2×10^{30} kg

Earth " " 6×10^{24} kg

1 AU = 1.5×10^{11} m

The Sun's radius is 7×10^{8} m

Is the Sun-Earth center of mass inside the Sun or outside?